

PHYS-3203 Homework 7 Due 11 Mar 2020

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. Chain Falling from a Table *inspired by problems from TM and FC*

A chain of linear mass density μ and length L lies on a table with one end hanging over the end. The chain slides off the table frictionlessly. The net force on the chain is the gravitational force on the length hanging off the table because the gravity acting on the chain lying on the table is cancelled by the normal force.

- Find the center of mass and total momentum of the chain when a length x is hanging vertically off the table and sliding off at rate \dot{x} . The chain that is still on the table is coiled up, so it does not move until it leaves the table.
- If the coiled chain starts falling off the table at $t = 0$ (ie, $x(0) = 0$), show that the chain experiences constant acceleration $\ddot{x} = g/3$.
- Find the total energy of the initially coiled chain at the moment it completely uncoils (that is $x = L$). How much energy is lost? This process is nonconservative because the uncoiling of each link is an inelastic process.
- Suppose that the chain is initially stretched out linearly on the table, so all the chain moves at the same speed as it slides off the table. In this case, the motion of the chain is elastic. Use energy conservation to find the speed of the chain when $x = L$ assuming it is at rest for $x = 0$.

2. Rutherford Scattering with Recoil

In PHYS-3202, we discussed scattering a particle of charge q_1 and mass m_1 from one of charge q_2 and mass m_2 with $m_2 \gg m_1$. In this $m_2 \rightarrow \infty$ limit, the charge q_2 does not move during the scattering. In this case, we found that the impact parameter b and differential cross section are

$$b = \frac{q_1 q_2}{4\pi\epsilon_0 m_1 v^2} \cot(\Theta/2), \quad \frac{d\sigma}{d\Omega} = \frac{1}{4} \left(\frac{q_1 q_2}{4\pi\epsilon_0 m_1 v^2} \right)^2 \frac{1}{\sin^4(\Theta/2)} \quad (1)$$

in terms of the asymptotic speed v of charge q_1 and the scattering angle Θ (the angle between initial and final momenta of charge q_1). In this problem, consider Rutherford scattering with finite m_2 , so the charge q_2 also moves.

- Show that equation (1) is true in the CM frame with the replacement of m_1 by the reduced mass μ and v by the magnitude of the asymptotic relative velocity. (In this frame we normally denote Θ as θ^* and solid angle Ω as Ω^* .)
- Now consider the Rutherford scattering of two charges with equal mass $m_1 = m_2 = m$ in a lab frame where charge q_2 is initially at rest. Find the relationship between the impact parameter and lab frame scattering angle θ and the lab frame differential cross section $d\sigma/d\Omega$ in terms of θ . Your answers will also depend on the initial speed of charge q_1 ; write your answer in terms of the mass m .