PHYS-3203 Homework 6 Due 4 Mar 2020

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Zero Mode** *From TM*

   Three coupled oscillators have the same mass m and potential energy

   $$V = \frac{k_1}{2}(x_1^2 + x_3^2) + \frac{k_2}{2}x_x^2 + \sqrt{\frac{k_1 k_2}{2}}(x_1 x_2 + x_2 x_3) \ . \tag{1}$$

   (a) Find the normal mode frequencies. *Hint:* One should be zero.

   (b) Find the (unnormalized) normal mode for the zero frequency.

   (c) Solve the equation of motion for the zero frequency normal coordinate. What does this represent? What is the potential energy of this normal coordinate? *Hint:* Given the frequency, we know the normal coordinate EOM; you do not need to find the normal coordinate in terms of the x coordinates.

2. **Alternating Masses** *from TM*

   Consider a light string loaded with massive beads with uniform spacing l between the beads where the beads at odd positions have mass m and at even positions have mass M. (That is, the first bead from the fixed end has mass m, the next M, and the next m, etc.) There is tension F along the entire string. Assume that both ends of the string are fixed to a wall and that the total number N of beads is odd.

   (a) Study transverse oscillations by guessing a normal mode solution $y_{2j+1} = a_1 \sin[(2j+1)\gamma]\exp(i\omega t)$, $y_{2j} = a_2 \sin(2j\gamma)\exp(i\omega t)$. Then show that the EOM can be written as an eigenvalue problem for the frequency, where $[a_1, a_2]^T$ is the eigenvector.

   (b) Using the boundary conditions, find the allowed values of γ. Solve the eigenvalue problem you found above and show that there are two normal mode frequencies for each allowed value of γ.

   (c) Consider the limit that $m \ll M$. Find the normal mode frequencies to first order in $1/M$ and unnormalized normal mode vectors to lowest nonvanishing order in $1/M$. Show that the lower frequency is the same as the frequency of a loaded string with just the mass M beads and describe the motion of the string for both normal modes.

3. **Vibration of String from Initial Conditions**

   Consider transverse oscillations of a string of linear mass density μ stretched between two supports a distance L apart with tension F. The phase velocity of waves on the string is $v = \sqrt{F/\mu}$.

   (a) *from KB 10.14* Suppose the string is plucked so that its initial configuration is two linear segments from ends to the midpoint, which has displacement a. The initial velocity of the string is zero. Describe the subsequent motion of the string and sketch its shape at times $t = L/4v, L/2v, L/v$.

   (b) Instead, suppose the string is struck with a sharp hammer blow at $x = L/2$, so the initial displacement is zero but the initial velocity is $\dot{y}(x,0) = Lv_0\delta(x - L/2)$ for some velocity v_0.

Describe the subsequent motion of the string. Is this type of motion physically possible for the string? *Hint:* remember that the integral of $\delta(x)$ is the Heaviside step function $\Theta(x)$.