PHYS-3203 Homework 3 Due 29 Jan 2020

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Falling Ladder** *from KB 10.12*

   A straight ladder of length $2L$ leans against a wall — one end is on the floor $y = 0$, and the other is on the wall $x = 0$. Both the wall and floor are frictionless. The ladder is symmetric, so its center of mass is at a distance L from either end, and it has mass M and moment of inertia I around the axis perpendicular to the xy plane and through the center of mass.

   (a) Find the position of the center of mass as a function of the angle θ of the ladder from the horizontal, assuming the end of the ladder is still touching the wall. Then write a Lagrangian for the motion of the ladder, the Euler-Lagrange equation, and energy conservation equation in terms of the generalized coordinate θ. *Hint:* remember that the kinetic energy can be written as translational kinetic energy of the center of mass plus rotational kinetic energy around the center of mass.

   (b) Now write the Lagrangian for the system by implementing the relationship between the center of mass positions x, y and θ with Lagrange multipliers. Find the equations of motion.

   (c) If the ladder is initially at rest at some angle θ_0, at what angle θ does it lose contact with the wall? Recall that the Lagrange multiplier gives the force of constraint. *Hint:* use the EOM for x re-written in terms of θ and then use the EOM and energy conservation you found in (a).

2. **Spring on a Pulley** *from KB 12.4 and Taylor*

   A light string of length l_1 hangs over a light pulley. On one side of the string is a mass m from which is hung a light spring with another mass m on the other side of the spring. On the other side of the pulley, the string supports a mass $2m$. The spring has spring constant k and equilibrium length l_2 (with mass m hanging from it).

   (a) Define x as the distance of the mass $2m$ from the pulley and y as the displacement of the spring from equilibrium (both increasing downward). Find the Hamiltonian in terms of x, y and their canonical momenta.

   (b) You should see that x is a cyclic coordinate. What does this fact represent?

   (c) Find Hamilton's equations. Solve for $x(t), y(t)$ if the mass $2m$ is initially at rest and the spring is stretched a small amount from equilibrium and released from rest. How does the frequency of oscillation compare to what it would be if the spring were hung from a stationary support?