Neutrino Oscillations

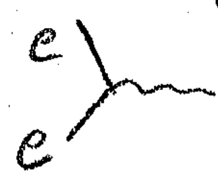
A little diversion to celebrate the Nobel Prize since we know enough

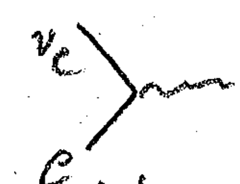
Flavor States vs. Mass States

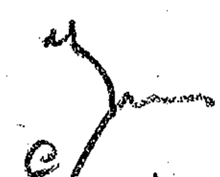
- What is flavor?
  - Subatomic "matter" particles come in 3 families, which are copies of each other except for having different masses
  - The electron is in the 1st family w/ its "sibling" the electron neutrino. The other families have the muon + mu neutrino and the tau and tau neutrino

$$\begin{array}{c|c} e & \nu_e \\ \mu & \nu_\mu \\ \tau & \nu_\tau \end{array} \qquad \begin{array}{l} m_e < m_\mu < m_\tau \\ m_\nu\text{'s} = ? \end{array}$$

  - Standard Model interactions create or destroy flavor eigenstates



or


etc, not


(at least not for these)

This is how we are conditioned to think of particles

- But what about the Hamiltonian?
  - We can think of the neutrino Hamiltonian as 2 parts: kinetic + interactions (like kinetic + potential).
  - We have $H = H_K(\nu) + H_I(\nu, \text{other stuff})$. The "flavor states" of neutrinos are eigenstates of H_I, <u>not</u> H.
  - The eigenstates of H_K are the "mass eigenstates"
- We are going to let the neutrinos be created as <u>flavor eigenstates</u> and then let them evolve according to H_K.
  - The key is that the mass eigenstates are different

$$|\nu_a\rangle = \sum_i U_a{}^i |\nu_i\rangle \quad (*), \qquad a = 1,2,3 = \text{mass state}; \quad i = e, \mu, \tau = \text{flavor state}$$

  - Take a simple example w/ 2 flavors (ie, 2D Hilbert space)

- The Kinetic Hamiltonian + Time evolution.
  - Neutrinos move relativistically, so we need a relativistic Hamiltonian
    - From special relativity, kinetic + rest energy is
      $H_K = \sqrt{(\vec{p}c)^2 + m^2c^4}$ for a single particle.
      + Normally, we do a <u>nonrelativistic</u> approximation
        $H_K \simeq mc^2 + \vec{p}^2/2m + \ldots$
      + For very energetic particles, $\vec{p}^2 \gg m^2c^2$, so
        $H_K \simeq |\vec{p}|c + m^2c^3/2|\vec{p}| + \ldots$. Neutrinos fit this case.
    - What if we have 2 (or more) neutrino states? + Use a column representation $|\Psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$
      + The Hamiltonian becomes a matrix in the <u>mass</u> <u>basis</u>.
        $$H_K = |\vec{p}|c\,\mathbb{1} + \frac{c^3}{2|\vec{p}|}\begin{bmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{bmatrix}$$
  - Time evolution is exactly what we've studied (consider a fixed momentum $\vec{p}$)
    - Mass/H_K eigenstates evolve by $e^{-iEt/\hbar}$
      + $|\nu_1\rangle \xrightarrow[\text{by } t]{\text{evolve}} e^{-ipct/\hbar}\, e^{-ic^3m_1^2t/2p\hbar}|\nu_1\rangle$, etc
      + In matrix form, $\begin{bmatrix} a \\ b \end{bmatrix} \rightarrow e^{-ipct/\hbar}\begin{bmatrix} a\exp(-ic^3m_1^2t/2p\hbar) \\ b\exp(-ic^3m_2^2t/2p\hbar) \end{bmatrix}$
    - Our initial state is a <u>flavor</u> state. We should write this in the mass basis.
      + In matrix form, we can write (✱) as $[\;]_{mass} = U[\;]_{flavor}$
      + For 2 neutrinos, the unique allowed matrix is
        θ = "mixing angle" $\rightarrow$ $U = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ ↑ some subtlety here, actually.
      + If we start with an electron neutrino, $|\nu_e\rangle \simeq \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{flavor}$,
        the initial state is $|\Psi(0)\rangle \simeq \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}_{mass}$.

- We immediately find $|\Psi(t)\rangle \simeq e^{-ipct/\hbar} \begin{bmatrix} \cos\theta \exp(-ic^3 m_1^2 t/2p\hbar) \\ -\sin\theta \exp(-ic^3 m_2^2 t/2p\hbar) \end{bmatrix}$

+ The probability of finding a ν_μ after a time t is

$$P_\mu = |\langle \nu_\mu | \Psi(t)\rangle|^2 = \left| [\sin\theta \;\; \cos\theta] \begin{bmatrix} \cos\theta \exp(\text{---}) \\ -\sin\theta \exp(\text{---}) \end{bmatrix} \right|^2$$

$$= \sin^2\theta \cos^2\theta \left| e^{ic^3 m_1^2 t/2p\hbar} - e^{-ic^3 m_2^2 t/2p\hbar} \right|^2$$

$$= \sin^2(2\theta) \sin^2\left[\frac{(m_2^2 - m_1^2) c^3 t}{4p\hbar} \right]$$

+ The neutrinos are moving basically at light speed, so
$ct = L =$ distance traveled, $pc = E =$ energy.
The famous formula is $P_\mu = \sin^2(2\theta) \sin^2\left[\frac{\Delta m^2 c^3 L}{E\hbar}\right]$

- Generalizations
  - For 3 neutrinos, U get's bigger + more complicated. There are 3 mixing angles + a complex phase.
  - If the neutrinos fly through matter, the average interactions generate a potential energy. • So even after the neutrino is created, we care about $H = H_K + V$
    - The potential V is diagonal in the flavor basis.
    - This effect can enhance oscillations a lot.