PHYS-4601 Homework 9 Due 22 Nov 2018

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Orthonormality of Harmonic Oscillator Eigenstates**

   Consider the eigenstates of the harmonic oscillator. *Hint:* This problem will be easier if you use techniques and results from previous homework assignments. You may also find the Gaussian integral formulas on the back cover of the text useful.

   (a) The Hermite polynomials satisfy the *Rodrigues formula*

   $$H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n} e^{-\xi^2} \ . \tag{1}$$

   Use this to show that H_n is an nth order polynomial of the form $H_n(\xi) = 2^n \xi^n + \cdots$. Then show that the eigenstate wavefunctions

   $$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2} \quad (\xi = \sqrt{\frac{m\omega}{\hbar}}\, x) \tag{2}$$

   are orthonormal (with the usual L^2 inner product).

   (b) Now define the energy eigenstates using the raising operator, $|n\rangle = (a^\dagger)^n |0\rangle/\sqrt{n!}$. First show that $[a, (a^\dagger)^n] = n(a^\dagger)^{n-1}$. Then show using operator techniques that these states are orthonormal (ie, $\langle n'|n\rangle = \delta_{n,n'}$).

2. **Former Term-Test Question 1**

   Two Hermitian operators A and B have simultaneous eigenstates denoted $|a, b\rangle$, where a is the eigenvalue of A and b is the eigenvalue of B. Answer the following questions as True or False. Explain your reasoning in one sentence per part.

   (a) The state $(|a_1, b\rangle + |a_2, b\rangle)/\sqrt{2}$ with $a_1 \neq a_2$ is an eigenstate of A.

   (b) The state $(|a_1, b\rangle + |a_2, b\rangle)/\sqrt{2}$ with $a_1 \neq a_2$ is an eigenstate of B.

   (c) The vector $(|a_1, b\rangle + 4|a_2, b\rangle)/3$ is correctly normalized.

3. **Former Term-Test Question 2**

   Some operator Q commutes with the Hamiltonian. If the initial state $|\Psi(t = 0)\rangle$ of the system is an eigenstate of Q with eigenvalue q, prove that $|\Psi(t)\rangle$ is also an eigenstate of Q with the same eigenvalue for any time t.

4. **Former Term-Test Question 3**

   Consider a particle of mass m in the infinite square well as defined on the cover sheet. Suppose the normalized wavefunction is

   $$\psi(x) = \frac{1}{\sqrt{2a}} \left[\sin\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi x}{a}\right) + 2\cos\left(\frac{\pi x}{2a}\right) \sin^2\left(\frac{\pi x}{2a}\right)\right] \ . \tag{3}$$

   What is the probability that a measurement of the energy yields $E = (\hbar^2\pi^2/8ma^2)$? *Hint:* You may find angle addition formulas to be helpful.