PHYS-4601 Homework 8 Due 8 Nov 2018

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Oscillating Square Well**

   Consider an infinite square well with potential $V(x) = 0$ for $-a < x < a$, $V(x) = \infty$ otherwise. The system initially has wavefunction

   $$\psi(x) = \frac{1}{2}\sqrt{\frac{3}{a}}\cos\left(\frac{\pi x}{2a}\right) - \frac{i}{2}\sqrt{\frac{1}{a}}\cos\left(\frac{5\pi x}{2a}\right) \tag{1}$$

   for $-a < x < a$ and $\psi(x) = 0$ otherwise.

   (a) If you measure the energy of the system, what possible energy values could you measure? And what are the probabilities of measuring them?

   (b) What is the probability density for finding the particle at position $x = 2a/3$ as a function of time?

   (c) At what times is the time-dependent wavefunction $\Psi(t, x)$ the opposite of the initial wavefunction, ie, when is $\Psi(t, x) = -\psi(x)$? (Note that this is physically equivalent to the initial wavefunction.)

   (d) In this part, suppose the system has different initial conditions

   $$\psi(x) = \sqrt{\frac{3}{2a}}\left(1 - \frac{|x|}{a}\right) \tag{2}$$

   for $-a < x < a$ and $\psi(x) = 0$ otherwise. That is, the wavefunction increases linearly from $\psi = 0$ at $x = -a$ to the origin, then decreases linearly to $\psi = 0$ at $x = a$ in a symmetric fashion. Based on your answer to the previous part, at what times t does the time-dependent wavefunction $\Psi(t, x) = -\psi(x)$? *Hint:* All you need to know is that this is an even wavefunction and the properties of the even bound states.

2. **Matrix Elements and Probabilities**

   (a) Calculate the matrix elements $\langle n|x|n'\rangle$ and $\langle n|p^2|n'\rangle$ for $|n\rangle, |n'\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals. **You may use $|m\rangle$ instead of $|n'\rangle$, but note that m here is a label, not the mass.**

   (b) Suppose the system is in the state $|\psi\rangle = (|0\rangle + 2e^{i\theta}|1\rangle)/\sqrt{5}$. Using your previous result, find $\langle x\rangle$ as a function of θ and explain the relation of your answer to the time evolution of a particle initially in that state with $\theta = 0$.

   (c) Now find the probability density for finding a particle in state $|\psi\rangle$ at position $x = 0$ as a function of θ.

3. **Wavefunctions and Ladder Operators**

   (a) *from Griffiths 2.10* Griffiths uses the condition $a|0\rangle = 0$ to find the ground state wavefunction, then $a^\dagger$ to find the first excited state wavefunction. Apply the raising operator $a^\dagger$ to $|1\rangle$ to find the wavefunction $\langle x|2\rangle$ of the second excited state.

(b) Prove that the Hermite polynomials satisfy the relationship

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - 2n H_{n-1}(\xi) \ . \tag{3}$$

Hint: consider $\langle x|x|n\rangle$ for the harmonic oscillator written in two different ways and then translate that in terms of wavefunctions.