PHYS-4601 Homework 16 Due 28 Feb 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **2-Qbit Gates**

   Consider a 2 qbit system. Choose a basis for the 2 qbit Hilbert space and use it for all parts of this problem.

   (a) Write the CNOT gate operator as a matrix in that basis and show that it is unitary.

   (b) Consider the 1 qbit gate NOT acting only on the first qbit of our two. Write this gate (call it NOT_1) as a matrix in your 2-qbit basis.

   (c) *inspired by Blümel 7.5.4* We can create a new quantum gate G by first acting with the NOT_1 and then CNOT. Give an example of an input 2-qbit state that can be factorized (that is, written as $|\psi\rangle_1|\phi\rangle_2$ for some 1-qbit states $|\psi\rangle$, $|\phi\rangle$) that is turned into an entangled state by G ($G(|\psi\rangle_1|\phi\rangle_2)$ cannot be factorized).

2. **Cloning Means FTL Communication** *based on a problem by Wilde*

   Suppose that Alice and Bob are at two ends of an EPR/Bell experiment. In other words, they are at rest with respect to each other and separated by 5 lightyears, and each receives one of a pair of entangled electrons with total spin state $s = 0$ simultaneously (in their common rest frame). By prior agreement, Alice measures either the S_z or S_x spin of her electron as soon as she receives it, but Bob does not know which spin she measures.

   After Alice's measurement (in their rest frame time), Bob's electron is in some state $|\psi\rangle_B$. Suppose, in contradiction to the no-cloning theorem, Bob can clone his electron's state onto a large number N of other electrons. (For example, Bob can do some quantum operation that takes his $N+1$ electrons from state $|\psi\rangle_B|\uparrow\rangle_1 \cdots |\uparrow\rangle_N$ to state $|\psi\rangle_B|\psi\rangle_1 \cdots |\psi\rangle_N$.) What measurement(s) can Bob do on his extra N electrons that will tell him with great certainty whether Alice measured the S_z or S_x spin of her electron? Explain your answer. (Note that Bob can accomplish his measurement before Alice can tell him her measurement choice, so they can establish faster-than-light communication in this way. This is a good reason for the no-cloning theorem!)

3. **Entanglement and Gates**

   (a) Start with two qbits in the state $|0\rangle|0\rangle$. Act on the first qbit with the Hadamard operator $\mathbb{H}$ and then use the CNOT operator. What is the total state $|\psi\rangle$? Is it entangled (a state is entangled if you cannot factorize it into a state for the first qbit times a state for the second)?

   (b) Act with the Hadamard operator on each qbit of the state $|\psi\rangle$ you found in the previous part. That is, find $\mathbb{H}_1\mathbb{H}_2|\psi\rangle$. Is it still entangled?

4. **Former Term-Test Question 1**

   As a simple model of a ferromagnetic material, consider 3 electrons at 3 different lattice sites in a circle, so they can be treated as distinguishable particles. The neighboring electron spins interact, and they have Hamiltonian

$$H = -\frac{E_0}{\hbar^2}\left(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1\right) , \tag{1}$$

where E_0 is a constant with dimensions of energy and $\vec{S}_i$ is the spin of electron i. Find the energy eigenvalues and give a brief argument why a small magnetic field can align all the spins.

5. **Former Term-Test Question 2**

   An electron in a hydrogen atom is in the combined space-spin state

$$\sqrt{\frac{2}{3}}|4,1,1\rangle|\uparrow\rangle - \frac{1}{\sqrt{3}}|3,1,-1\rangle|\downarrow\rangle \ . \tag{2}$$

   Let $\vec{J}$ be the total angular momentum operator for the electron. List the possible values you can measure for J_z and their probabilities. Then list the possible values and the probabilities for J^2.