PHYS-4601 Homework 15 Due 7 Feb 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Comparing Expectation Values**
   (a) *based on Griffiths 4.13* Find $\langle r^2 \rangle$ for the ground state of a hydrogen-like atom (a single electron moving in a central Coulomb potential) in terms of the Bohr radius. Find the ratio of this result between hydrogen to that for a helium ion, which has a single electron orbiting a nucleus of charge $+2e$. (In other words, find $\langle r^2 \rangle_H / \langle r^2 \rangle_{He^+}$.) What does this mean about the comparative "size" of these two atoms?
   (b) Now find the ratio of $\langle r^2 \rangle$ for the $n = 2, \ell = 1, m = 0$ state of hydrogen to the $n = 2, \ell = 0, m = 0$ state.
   (c) Finally, find the ratio of $\langle z^2 \rangle$ for the $n = 2, \ell = 1, m = 0$ state of hydrogen to the $n = 2, \ell = 0, m = 0$ state. *Hint:* You can find $\langle z^2 \rangle$ for the $n = 2, \ell = 0, m = 0$ state by using symmetry arguments and your work from part (b).

2. **The GHZM Experiment**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture at `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF` . This is the video listed on the reading assignment.

   Three electrons are prepared in the so-called "GHZM" spin state $|\psi\rangle = (|+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3)/\sqrt{2}$ and distributed so that laboratories at locations A, B, and C each receive one electron.
   (a) Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.
   (b) If the electrons' total state is written as $|\phi\rangle \otimes |\psi\rangle$, where $|\phi\rangle$ is the spatial state of the electrons, what is $|\phi\rangle$? Each single-electron spatial state is $|A\rangle$, $|B\rangle$, or $|C\rangle$.

3. **Many Worlds Quantum Mechanics**

   You are alone in a closed laboratory, and you possess a sodium atom with positive spin oriented along the $+x$ axis (the sodium ground state has total angular momentum $J = 1/2$). You then send the atom through a Stern-Gerlach apparatus and measure the z component of the atom's angular momentum. In the many worlds interpretation of quantum mechanics, what is the state of the system (a) before and (b) after your measurement?