PHYS-4601 Homework 14 Due 31 Jan 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Quantum Earth** *almost Griffiths 4.17*

   In this problem, treat the earth-sun system as an analog of the hydrogen atom. Let M be the mass of the sun and m the mass of the earth.

   (a) By comparing the Newtonian gravitational potential to the Coulomb potential of the hydrogen atom, write down the gravitational Bohr radius a_g and the quantum mechanical earth-sun energy E_n in terms of M, m, and the Newton constant G.

   (b) Compare the classical energy of a planet in a circular orbit of radius r to your formula E_n and show that $n = \sqrt{r/a_g}$. Estimate n for the earth. Let r be 1 astronomical unit. Just give one significant digit. *Hint*: You can look up all the astrophysical data you need at `http://pdg.lbl.gov/2018/reviews/rpp2018-rev-astrophysical-constants.pdf`. It also helps to remember the virial theorem, which says that kinetic energy is $-1/2$ potential energy for an orbit in a $1/r$ potential.

   (c) Show that the total orbital angular momentum quantum number ℓ is approximately n; that is, ℓ is close to its maximum allowed value.

   (d) *related to Griffiths 4.46* We can now relate our estimated results to quantum mechanics. Start by using the recursion relation discussed in class for the radial wavefunction to show that the radial wavefunction for a state with $\ell = n-1$ is $R \propto r^{n-1}\exp[-r/na_g]$. Find the normalization constant and then $\langle r\rangle$. For large n, show that this agrees with your previous result $n = \sqrt{r/a_g}$.

2. **States of Baryons**

   Baryons are particles made up of three *quarks*, which are themselves spin-1/2 particles. In addition to spin, these quarks possess quantum "numbers" called *flavor* and *color*. A single quark's color has three basis states, red ($|R\rangle$), green ($|G\rangle$), and blue ($|B\rangle$), and the quarks in light baryons have three possible flavors, up ($|u\rangle$), down ($|d\rangle$), and strange ($|s\rangle$). Assume that the ground state for any baryon has a wavefunction that is symmetric under exchange of any of the quarks.

   (a) The Δ^{++} baryon has spin-3/2, and its charge (+2) means it is composed of 3 up quarks. Therefore, the $m_s = +3/2$ state has flavor-spin state $|u\rangle_1|u\rangle_2|u\rangle_3|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$. Write a possible color state for the three quarks.

   (b) Due to the strong nuclear force, all baryons actually must have the same color state (your answer to part (a)). Now consider the Σ^{*-} particle, which is also spin-3/2 and is composed of two down quarks and one strange quark. The $m_s = +3/2$ state has spin state $|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$; write the flavor state of this baryon.

3. **Orbitals and Electrons**

   (a) Beryllium has electronic structure $(1s)^2(2s)^2$. What is the total spin quantum number s of the two $2s$ electrons?

(b) Recall that the single particle stationary states of an infinite square well of length L have energy eigenvalues $E_n = (\hbar^2\pi^2/2mL^2)n^2$ for $n = 1, 2, \cdots$. If two electrons are placed in the square well, describe the ground state, first excited state, and second excited state, including both spatial and spin components and their degeneracies (for example, if the ground state has degeneracy 3, list the next two energy levels also). Assume that the Coulomb repulsion between electrons is nonnegligible but small compared to the spacing between square well energy levels.