PHYS-4601 Homework 13 Due 24 Jan 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Hadron Spins** *Griffiths 4.35 plus*

   Quarks are elementary particles with spin 1/2, which we see in bound states called *hadrons*. Hadrons come in two varieties. In the following, assume that the quarks have zero orbital angular momentum.

   (a) *Mesons* are formed of a quark and antiquark (think of it as two quarks). What are the possible total spin quantum numbers of a meson?

   (b) *Baryons* are formed of three distinct quarks. What are the possible total spin quantum numbers? How many complete sets of states are there for each of those total spins?

2. **Spin Interactions**

   (a) Two spin 1/2 particles are fixed in position but have interacting spins. Their Hamiltonian is

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} \tag{1}$$

   for some constant J. Here $S^{(i)}$ is the spin operator of the ith particle. Find the energy eigenvalues of this system, their degeneracies, and the corresponding eigenstates. *Hint*: You will want to work in terms of the total spin quantum numbers.

   (b) The two spins have the same gyromagnetic ratio γ. In the presence of a magnetic field, the Hamiltonian becomes

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} - \gamma\vec{B} \cdot \left(\vec{S}^{(1)} + \vec{S}^{(2)}\right) . \tag{2}$$

   Now find the energy eigenvalues and their degeneracies. You may take $\vec{B}$ to lie along the z direction.

   (c) Suppose that initially the first spin is up, while the second spin is down (ie, $|+\rangle$ and $|-\rangle$ respectively). What is the probability of finding the first spin up at a later time t? If both spins are initially up, what is the probability?

3. **Change of Basis**

   Consider an electron (spin-1/2) moving in a hydrogen atom. *Hint:* You may find the Clebsch-Gordan coefficient table and equation (1) from assignment 11 helpful.

   (a) Suppose the electron has orbital angular momentum $\ell = 2$. Write the total angular momentum $j = 5/2$, $m_j = 3/2$ state in terms of the orbital and spin angular momentum states. That is, write it as a sum of the form $\sum c_{\ell,m,s,m_s}|\ell, m\rangle|s, m_s\rangle$.

   (b) Assume $l = 2$ again, and suppose that the z components of orbital angular momentum and spin are given by $m = 0, m_s = +1/2$. If you measure the total angular momentum, what is the probability of measuring $m_j = +3/2$? $m_j = +1/2$? $j = 5/2$? $j = 3/2$?

   (c) Suppose instead that $l = 3$. Now write the total angular momentum $j = 7/2$, $m_j = -5/2$ state in terms of the orbital and spin angular momentum states.