PHYS-3301 Winter Homework 2 Due 24 Jan 2018

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Some Counting Questions**

   (a) Consider expanding $(a+b+c)^N$ as a multinomial. Show that the coefficient of $a^{N_1}b^{N_2}c^{N_3}$ is the *multinomial coefficient* $N!/N_1!N_2!N_3!$. *Hint:* you may want to make an analogy to distinguishable particles and energy levels.

   Consider the example of 5 particles in a harmonic oscillator from the class notes. As in the notes, assume the particles have total energy $25\hbar\omega/2$, corresponding to a total excitation number of 10.

   (b) In class, we assumed the particles were distinguishable. If the particles are instead identical fermions, how many ways are there to arrange 5 fermions with total excitation number 10?

   (c) In class, we said that there are 30 ways to arrange 5 distinguishable particles with occupancy numbers $N_0 = 1, N_2 = 2, N_3 = 2$ (with all others vanishing). If the particles are instead identical bosons, how many ways are there to arrange the 5 particles with those occupancy numbers?

2. **The Ideal Gas**

   A classical ideal gas is composed of molecules of mass m with the number of molecules per energy level given by the Maxwell-Boltzmann distribution for distinguishable particles. As discussed in class, the number of particles in a state of energy E_k is

   $$n(E_k) = e^{(\mu - E_k)/k_BT} \text{ where } E_k = \frac{\hbar^2 k^2}{2m} \ , \tag{1}$$

   and the degeneracy at energy E_k is $d_k = (V/2\pi^2)k^2 dk$, as discussed in the class notes. You may find the Gaussian integral formulae in the back cover of Griffiths to be useful.

   (a) Show that the number of air molecules is given by $N/V = (mk_BT/2\pi\hbar^2)^{3/2}\exp(\mu/k_BT)$.

   (b) Show that the total energy is $3Nk_BT/2$.

3. **Cold Electron Gas**

   Electrons confined in a piece of metal can also be described as particles in a 3D infinite square well. They have 2 spin states, so their energy levels have twice the degeneracy of an ideal gas $d_k = (V/\pi^2)k^2 dk$. They follow the Fermi-Dirac distribution; as $T \to 0$, this distribution turns in to a step function that is equal to 1 for $E_k < \mu$ and 0 for $E_k > \mu$. Find the relationships between the number N of electrons and μ and between total energy E and μ.

4. **Stefan-Boltzmann Formula** *like Griffiths 5.31 (and in many other places)*

   Now we consider photons following the Planck law discussed in class. Integrate the Planck distribution over all frequencies to show that the energy density of light at temperature T is $E/V = \alpha(k_BT)^4/(\hbar c)^3$, where α is a numerical constant (you do not need to evaluate α but can leave it in the form of a dimensionless integral). This is one form of the Stefan-Boltzmann formula, which was actually known before the Planck law.