PHYS-3301 Winter Homework 10 Due 28 Mar 2018

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

   $$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 . \qquad (1)$$

   Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

   (b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E. Use (1).

   (c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.

   For your interest, these neutrino measurements were made by a predecessor experiment to one of the experiments that led to the 2015 Nobel Prize in Physics.

2. **Large Hadron Collider**

   The Large Hadron Collider (LHC) collides pairs of protons with a total energy of about 10 TeV ($= 10^4$ GeV) in their CM frame. For this problem, you will want to know that the mass of a proton is about 1 GeV/c^2.

   (a) What is the energy of one of the protons as measured in the rest frame of the other proton? *Hint:* First show that $(p_1 + p_2)^2$, where p_1^μ, p_2^μ are the 2 initial 4-momenta, gives the total CM frame energy. Then show that the relativistic scalar product $p_1 \cdot p_2$ gives the energy of the first proton in the 2nd proton's rest frame and relate the two products.

   (b) Suppose the LHC observes a collision that produces two photons and a number of other particles. The two photons have energies of 0.9 TeV and 0.6 TeV, and their paths are at an angle of 60 degrees to each other. If the collision process can be described as 2 protons become particle X (plus other particles we don't care about), followed by particle X decaying into the 2 photons, what is the mass of particle X?

3. **Neutrino Recoil**

   When a muon-neutrino and an electron collide, the neutrino and electron can transform into an electron-neutrino and a muon. For reference, the electron and muon have masses m_e and m_μ respectively with $m_\mu > m_e$, and you may approximate both types of neutrino as being massless.

   (a) If the electron is initially at rest, what is the minimum initial neutrino energy required for this process to occur?

   (b) Suppose the electron is initially at rest and the final electron-neutrino moves off at an angle θ from the initial direction of motion of the muon-neutrino. If the initial muon-neutrino energy is E, find the energy of the electron-neutrino.

4. **Annihilation in the Early Universe**

   In the rest frame of the cosmic fluid in the early universe, an electron moves in the $+x$ direction with energy E_1, and a positron moves along the $+y$ direction with energy E_2. They hit each other at the origin and annihilate to produce two photons (before the collision, there are an electron and positron; after there are two photons). One of the photons travels in the $-x$ direction.

   (a) What is the energy of that photon? Electrons and positrons have mass m.

   (b) What is your answer in the limit that $E_1 \gg E_2, m$? In the limit that $E_2 \gg E_1, m$?