Quantum Mechanics I PHYS-3301 In-Class Test

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Instructions:

- Do not turn over until instructed. You will have 50 minutes to complete this exam.
- No electronic devices or hardcopy notes are allowed.
- INSTRUCTIONS ABOUT THE QUESTIONS WILL GO HERE.
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Concepts & Formulae:

- Physical Constants
  - Speed of light $c = 3 \times 10^8$ m/s = 1 lightsecond/second = 1 lightyear/year
  - Planck constant $h = 2\pi\hbar = 7 \times 10^{-34}$ Js
  - Boltzmann constant $k_B = 10^{-23}$ J/K
- Quantum Mechanics
  - Spherical harmonics will be given if needed
  - Momentum space wavefunction $\tilde{\psi}(p)$ with $p \cdot \tilde{\psi} = p\tilde{\psi}$, $x \cdot \tilde{\psi} = i\hbar d\tilde{\psi}/dp$

$$\tilde{\psi}(p) = \int \frac{dx}{\sqrt{2\pi\hbar}} e^{-ipx/\hbar}\psi(x) \ , \quad \psi(x) = \int \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}\tilde{\psi}(p)$$

  - Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein distributions

$$n_{MB}(E) = e^{-(E-\mu)/k_BT} \ , \quad n_{FD}(E) = \frac{1}{e^{(E-\mu)/k_BT}+1} \ , \quad n_{BE}(E) = \frac{1}{e^{(E-\mu)/k_BT}-1}$$

  - Free particles: $E_k = \hbar^2\vec{k}^2/2m$ with degeneracy $d_k = (sV/2\pi^2)k^2dk$ where s =number of polarizations
- Galilean Relativity/Newtonian Mechanics
  - The CM frame is the frame in which the total spatial momentum is zero.
  - Rotations are given by multiplication with a rotation matrix.
  - Galilean boost $\vec{x}' = \vec{x} - \vec{v}t$, $\vec{u}' = \vec{u} - \vec{v}$, $\vec{p}' = \vec{p} - m\vec{v}$, $k' = k - \vec{p}\cdot\vec{v} + (1/2)mv^2$
  - Kinetic energy for many particles $K = K_{int} + (1/2)MV^2$

$$M = \sum m_i \ , \quad \vec{V} = \frac{1}{M}\sum m_i\vec{u}_i$$

  - For two particles $K_{int} = (1/2)\mu u^2$ for relative velocity $\vec{u}$ and reduced mass $\mu = m_1m_2/M$
- The Lorentz boost transformations (in standard configuration) are

$$t' = \gamma(t - vx/c^2) \ , \ x' = \gamma(x - vt) \ , \quad y' = y \ , \quad z' = z \ , \quad \gamma = 1/\sqrt{1 - v^2/c^2} \ .$$

- Math
  - Polar coordinates $d^3\vec{x} = r^2 \sin\theta dr d\theta d\phi$
    The region $-\infty < x, y, z < \infty$ is $0 < r < \infty, 0 < \theta < \pi, 0 < \phi < 2\pi$
  - Exponential integrals (for $n = 0, 1, \cdots$ and $a > 0$)

$$\int_0^\infty dx\, x^n e^{-x/a} = n! a^{n+1}$$

  - Gaussian integrals

$$\int_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}\ , \quad \int_{-\infty}^{\infty} dx\, x^2 e^{-ax^2} = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

  - Hyperbolic trig functions: $d\cosh\theta/d\theta = \sinh\theta$, $d\sinh\theta/d\theta = \cosh\theta$

$$\cosh^2\theta - \sinh^2\theta = 1\ , \quad \cosh^2\theta + \sinh^2\theta = \cosh(2\theta)\ , \quad 2\sinh\theta\cosh\theta = \sinh(2\theta)$$

  - Binomial expansion $(1+x)^n \approx 1 + nx$ for $x \ll 1$