PHYS-4601 Homework 9 Due 24 Nov 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Commutators and Things** *partly from Griffiths 4.19, partly inspired by problems in Ohanian*
   (a) Find the commutators $[L_z, x]$, $[L_z, y]$, $[L_z, p_x]$, and $[L_z, p_y]$.
   (b) Show that $[L_z, H] = 0$ for Hamiltonian $H = \vec{p}^2/2m + V$ with central potential V. Argue that therefore $[\vec{L}^2, H] = 0$ also. *Hint:* Assume $V(r)$ is a power series; then argue L_z and r commute based on $[L_z, r^2]$.
   (c) Using the commutation relations, find $\vec{L} \times \vec{L}$ (as an operator quantity). How does your result relate to the classical statement that a vector cross itself is zero?

2. **Probabilities and Expectations** *based on problems by Ohanian*

   Some particle has a wavefunction

   $$\psi(\vec{x}) = \frac{1}{4}\sqrt{\frac{5}{\pi}} \sin^2\theta \left[1 + \sqrt{14}\cos\theta\right] \cos(2\phi) R(r) \ , \tag{1}$$

   where R is normalized ($\int_0^\infty dr\, r^2 |R|^2 = 1$).
   (a) If you measure the total orbital angular momentum $\vec{L}^2$, what are the possible values you would find and the probabilities you would measure each value?
   (b) Find $\langle L_x \rangle$, $\langle L_y \rangle$, and $\langle L_z \rangle$ in this state.
   (c) Find the uncertainty of L_z.

3. **Angular Dependence of Spherical Harmonic Oscillator**

   Consider a 3D isotropic harmonic oscillator (potential $V(\vec{x}) = m\omega^2 r^2/2$). Recall from a previous assignment that separation of variables in Cartesian coordinates allows us to write the eigenstates as $|n_x, n_y, n_z\rangle = (1/\sqrt{n_x! n_y! n_z!})(a_x^\dagger)^{n_x}(a_y^\dagger)^{n_y}(a_z^\dagger)^{n_z}|0\rangle$ in terms of the Cartesian harmonic oscillator ladder operators ($a_x, a_x^\dagger$, etc). Write L_z in terms of the Cartesian harmonic oscillator ladder operators. Then use ladder operator techniques to show that $(|1,0,0\rangle \pm i|0,1,0\rangle)/\sqrt{2}$ and $|0,0,1\rangle$ are eigenstates of L_z and find their eigenvalues.

4. **Raising and Lowering**
   (a) *More or less Griffiths 4.18* Using the relation for $L_\pm L_\mp$ given in class and the text, show that

   $$L_\pm |\ell, m\rangle = \hbar\sqrt{(\ell \mp m)(\ell \pm m + 1)}|\ell, m \pm 1\rangle \ . \tag{2}$$

   (b) In a vector/matrix representation of the $\ell = 1$ states where

   $$|1,1\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ , \quad |1,0\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \ , \quad |1,-1\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ , \tag{3}$$

   use (2) to find matrix representations of $L_\pm$ and then L_x and L_y.
   (c) *Griffiths 4.22(b)* Use $L_+ \cdot Y_\ell^\ell = 0$ and $L_z \cdot Y_\ell^\ell = \ell\hbar Y_\ell^\ell$ to determine $Y_\ell^\ell(\theta, \phi)$ up to overall normalization.