PHYS-4601 Homework 7 Due 3 Nov 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Matrix Elements and Probabilities**
   (a) Calculate the matrix elements $\langle n|x|m\rangle$ and $\langle n|p^2|m\rangle$ for $|n\rangle, |n'\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals.
   (b) Suppose the system is in the state $|\psi\rangle = (|0\rangle + 2e^{i\theta}|1\rangle)/\sqrt{5}$. Using your previous result, find $\langle x\rangle$ as a function of θ and explain the relation of your answer to the time evolution of a particle initially in that state with $\theta = 0$.
   (c) Now find the probability density for finding a particle in state $|\psi\rangle$ at position $x = 0$ as a function of θ.

2. **Wavefunctions and Ladder Operators**
   (a) *from Griffiths 2.10* Griffiths uses the condition $a|0\rangle = 0$ to find the ground state wavefunction, then $a^\dagger$ to find the first excited state wavefunction. Apply the raising operator $a^\dagger$ to $|1\rangle$ to find the wavefunction $\langle x|2\rangle$ of the second excited state.
   (b) Prove that the Hermite polynomials satisfy the relationship

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - 2nH_{n-1}(\xi) \ . \tag{1}$$

   Hint: consider $\langle x|x|n\rangle$ for the harmonic oscillator written in two different ways and then translate that in terms of wavefunctions.

3. **Coherent States** *based on Griffiths 3.35 and beyond*
   In this problem, we will study *coherent states*, which are eigenfunctions of the lowering operator

$$a|\alpha\rangle = \alpha|\alpha\rangle \ , \tag{2}$$

   where the eigenvalue α is generally complex.
   (a) Is there any energy eigenstate that is a coherent state? If so, list which energy eigenstate(s) are coherent and give the eigenvalue(s).
   (b) Find the expectation values of x and p in the coherent state $|\alpha\rangle$ (use the ladder operators).
   (c) Then find the uncertainties of x and p in $|\alpha\rangle$ and show that any coherent state is a state of minimal undertainty.
   (d) Show that

$$|\alpha\rangle = e^{-|\alpha|^2/2} \exp\left[\alpha a^\dagger\right]|0\rangle \ . \tag{3}$$

   is a coherent state of eigenvalue α.
   (e) A *squeezed state* $|\zeta\rangle$ is a state that obeys the equation $a|\zeta\rangle = \zeta a^\dagger|\zeta\rangle$ for a complex number ζ with $|\zeta| < 1$. Show that $\langle x\rangle = 0$ and $\langle p\rangle = 0$ in the squeezed state $|\zeta\rangle$. *Hint:* Reduce the problem to showing that $\langle a\rangle = 0$. It will help to remember that $\langle a\rangle^* = \langle a^\dagger\rangle$.