PHYS-4601 Homework 4 Due 6 Oct 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Measurement vs Time Evolution** *a considerable expansion of Griffiths 3.27*

   Suppose a system has observable A with eigenstates $|a_1\rangle, |a_2\rangle$ of eigenvalues a_1, a_2 respectively and Hamiltonian H with eigenstates $|E_1\rangle, |E_2\rangle$ of energies E_1, E_2 respectively. The eigenstates are related by

   $$|a_1\rangle = \frac{1}{5}\left(3|E_1\rangle + 4|E_2\rangle\right) \ , \quad |a_2\rangle = \frac{1}{5}\left(4|E_1\rangle - 3|E_2\rangle\right) \ . \tag{1}$$

   Suppose the system is measured to have value a_1 for A initially. Each of the following parts asks about a different possible set of subsequent measurements.

   (a) What is the probability of measuring energy E_1 immediately after the first measurement? Assuming we do get E_1, what is the probability of measuring a_1 again if we measure A again immediately after the measurement of energy?

   (b) Instead, consider immediately measuring A again after the first measurement. What are the probabilities for observing a_1 and a_2?

   (c) Finally, consider making the first measurement and then allowing the system to evolve for time t. If we then measure energy, what is the probability of finding energy E_1? If we instead measured A again, what is the probability we find a_1 again?

2. **The Virial Theorem** *Based on Griffiths 3.31*

   Consider 3D quantum mechanics.

   (a) Using Ehrenfest's theorem, show that

   $$\frac{d}{dt}\langle \vec{x}\cdot\vec{p}\rangle = \left\langle \frac{\vec{p}^2}{m}\right\rangle - \left\langle \vec{x}\cdot\vec{\nabla}V(\vec{x})\right\rangle \ . \tag{2}$$

   (b) Show that the left-hand side of (2) vanishes in a stationary state to prove the *virial theorm*

   $$2\langle K\rangle = \left\langle \vec{x}\cdot\vec{\nabla}V(\vec{x})\right\rangle \ , \tag{3}$$

   where K is the kinetic energy. (The virial theorem holds classically, also, though without the expectation values.)

   (c) Using the virial theorem, find the ratio of (the expectation value of) the kinetic energy to the potential energy for the harmonic oscillator potential $V \propto \vec{x}^2$ and for the Coulomb potential $V \propto 1/|\vec{x}|$. *Hint:* Remember from electromagnetism (or Newton's law of gravity) that $\vec{\nabla}(1/|\vec{x}|) = -\vec{x}/|\vec{x}|^3$.

3. **Matrix Hamiltonian**

   Consider a 3D Hilbert space with Hamiltonian

   $$H \simeq E_0 \begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix} \tag{4}$$

   in some basis. Work in this basis throughout the problem.

(a) Show that the time evolution operator is

$$e^{-iHt/\hbar} \simeq \cos(E_0 t/\hbar) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - i \sin(E_0 t/\hbar) \begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix} \tag{5}$$

in this basis.

(b) Some operator A is defined in this basis as

$$A \simeq A_0 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} . \tag{6}$$

Suppose the system starts out at time $t = 0$ in a state represented by $[1\ 0\ 0]^T$. Using your previous result, find the state of the system and $\langle A \rangle$ as a function of time. At what times is $\langle A \rangle$ minimized?