

PHYS-4601 Homework 15 Due 16 Feb 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. White Dwarfs based on Griffiths 5.35

White dwarfs are old, dead stars that don't support nuclear fusion any more. Instead, electron degeneracy pressure keeps them from collapsing into black holes. In this problem, you'll work out the white dwarf radius by finding the minimum energy. *Note:* You will need equation [5.45] from Griffiths.

- Imagine that the white dwarf is a sphere of uniform density. Calculate the gravitational potential energy V in terms of its radius R , total mass M , and Newton's constant G . *Hint:* To do this, work out the potential energy of a shell of thickness dr at radius r , which comes from the sphere of matter inside that shell. Then integrate from $r = 0 \rightarrow R$. You should find $V = -3GM^2/5R$.
- Now treat the electrons as a free electron gas. Write their total energy (which is kinetic) in terms of M , R , the electron mass m , the nucleon mass m_N , the number of electrons per nucleon q , and \hbar . Then add this to the result of part (a) to get the total energy and find the radius R where the energy is minimized.
- Assuming $q \approx 1/2$ (the average nucleus is a helium nucleus), find the radius (in km) of a white dwarf with the mass of the sun. You will need $G = 7 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$, $\hbar = 1 \times 10^{-34} \text{ Js}$, $m = 9 \times 10^{-31} \text{ kg}$, $m_N = 2 \times 10^{-27} \text{ kg}$, and $M = 2 \times 10^{30} \text{ kg}$.
- A neutron star is an extremely dense star where all the matter is neutrons, which is supported against collapse by the degeneracy pressure of the neutrons. In this case, $q = 1$. Find the ratio of the radius of a neutron star to the radius of a white dwarf of the same mass (assume $q \approx 1$ for the white dwarf), both in terms of physical constants and as a pure number to 1 significant digit.

2. Bloch's Theorem

In class, we stated Bloch's theorem as saying that any stationary state wavefunction of a periodic potential $V(x+a) = V(x)$ can be written to satisfy the condition

$$\psi(x+a) = e^{iKa}\psi(x) \quad (K \text{ real}) . \quad (1)$$

Alternately, Bloch's theorem can state that any stationary state wavefunction of the same potential can be written as

$$\psi(x) = e^{iKx}u(x) \text{ where } u(x+a) = u(x) . \quad (2)$$

- Show that these two formulations are equivalent (that is, show that if ψ satisfies (1) then it satisfies (2) and vice-versa).
- If you write $u(x) = \sum_q c_q e^{iqx}$ as a Fourier series, what are the allowed values of q ? Therefore, what are the allowed values of the momentum for a wavefunction with a given K ?