PHYS-4601 Homework 13 Due 26 Jan 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Estimating the Helium Ground State** *Griffiths 5.11 clarified*

   In this problem, we will estimate the ground state energy of a helium atom. We treat the electron repulsion as a first-order correction to the attraction between the electrons and the nucleus.

   (a) Consider the states of a single electron around a helium nucleus (which has twice the charge of a proton). Argue that the "helium Bohr radius" $a_{\text{He}} = a/2$, where a is the usual Bohr radius, and that therefore the single-electron ground state wavefunction is given by

   $$\langle \vec{x} | n = 1, \ell = 0, m = 0 \rangle = \sqrt{\frac{8}{\pi a^3}} e^{-2r/a} \ . \tag{1}$$

   Next assume that the two electron helium groundstate is $|n = 1, \ell = 0, m = 0\rangle_1 |n = 1, \ell = 0, m = 0\rangle_2 |s = 0, m_s = 0\rangle$, where the total spin state is the antisymmetric singlet. (The spatial wavefunction is given by Griffiths eqn [5.30].) Briefly argue that the energy of this state, in the absence of electron repulsion, is given by Griffiths eqn [5.31].

   (b) Now find $\langle |\vec{x}_1 - \vec{x}_2|^{-1} \rangle$ in this state, as follows:

   1. Use the trick of setting the z axis for $\vec{x}_2$ along $\vec{x}_1$ and the law of cosines to see $|\vec{x}_1 - \vec{x}_2| = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta_2}$.
   2. Do the angular integrals for $\vec{x}_2$, noting that

      $$\int_0^\pi d\theta \sin\theta f(\cos\theta) = \int_{-1}^1 dx f(x) \ .$$

      Your result will have square roots of perfect squares, which are equal to absolute values. *Be careful of that!*
   3. Carry out the r_2 integral in two parts, $0 < r_2 \leq r_1$ and $r_1 < r_2 < \infty$.
   4. Now do the $\vec{x}_1$ integrals.

   Hint: The "exponential integrals" formula in the back cover of Griffiths will be helpful.

   (c) Use your result to find the change in ground state energy ΔE at first order in perturbation theory. Write ΔE in terms of the Bohr radius a and estimate its value in eV. Then add this to the energy from part (a) to get a rough estimate of the He ground state energy. *Hint*: Remember that the hydrogen ground state energy is $-\hbar^2/2ma^2 = -13.6$ eV.

2. **The GHZM Experiment**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture at `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF` . This is the video listed on the reading assignment.

   Three electrons are prepared in the so-called "GHZM" spin state $|\psi\rangle = (|+\rangle_1 |+\rangle_2 |+\rangle_3 - |-\rangle_1 |-\rangle_2 |-\rangle_3)/\sqrt{2}$ and distributed so that laboratories at locations A, B, and C each receive one electron.

   (a) Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.

(b) If the electrons' total state is written as $|\phi\rangle \otimes |\psi\rangle$, where $|\phi\rangle$ is the spatial state of the electrons, what is $|\phi\rangle$? Each single-electron spatial state is $|A\rangle$, $|B\rangle$, or $|C\rangle$.

3. **Many Worlds Quantum Mechanics**

   You are alone in a closed laboratory, and you possess a sodium atom with positive spin oriented along the $+x$ axis (the sodium ground state has total angular momentum $J = 1/2$). You then send the atom through a Stern-Gerlach apparatus and measure the z component of the atom's angular momentum. In the many worlds interpretation of quantum mechanics, what is the state of the system (a) before and (b) after your measurement?