PHYS-4601 Homework 12 Due 19 Jan 2017 2ND REVISION

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Multiple Particle Wavefunctions** *based on a problem in Ohanian*

   Consider two free spin 1/2 particles, which have single particle states $|\psi_1\rangle = |\vec{p}_1\rangle|+\rangle$ and $|\psi_2\rangle = |\vec{p}_2\rangle|-\rangle$. These states are factorized into spatial states (in this case, momentum eigenstates) and spin states (eigenstates of S_z).

   (a) Write the two-particle wavefunction if the two particles are distinguishable (say, particle 1 is a proton and particle 2 is an electron).

   (b) Now suppose that both particles are electrons, so they are indistinguishable. Write the two-particle state which is an eigenfunction of the total spin operator $\vec{S}^2$ with eigenvalue given by $s = 0$.

   (c) Keeping indistinguishable electrons, now write the two-particle wavefunction for with total spin eigenvalues $s = 1$, $m = 0$.

   (d) Finally, consider the case where the particles are indistinguishable but instead have spin 0, so there is no spin part of their states. Write the allowed two-particle state.

2. **States of Baryons**

   Baryons are particles made up of three *quarks*, which are themselves spin-1/2 particles. In addition to spin, these quarks possess quantum "numbers" called *flavor* and *color*. A single quark's color has three basis states, red ($|R\rangle$), green ($|G\rangle$), and blue ($|B\rangle$), and the quarks in light baryons have three possible flavors, up ($|u\rangle$), down ($|d\rangle$), and strange ($|s\rangle$). Assume that the ground state for any baryon has a wavefunction that is symmetric under exchange of any of the quarks.

   (a) The Δ^{++} baryon has spin-3/2, and its charge (+2) means it is composed of 3 up quarks. Therefore, the $m_s = +3/2$ state has flavor-spin state $|u\rangle_1|u\rangle_2|u\rangle_3|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$. Write a possible color state for the three quarks.

   (b) Due to the strong nuclear force, all baryons actually must have the same color state (your answer to part (a)). Now consider the Σ^{*-} particle, which is also spin-3/2 and is composed of two down quarks and one strange quark. The $m_s = +3/2$ state has spin state $|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$; write the flavor state of this baryon.

3. **Hydrogen Molecule**

   The electrons in a molecule are tightly bound and provide a potential for the motion of the nuclei. For a diatomic molecule, it is often a good approximation to expand the potential around the equilibrium separation r_0 as

   $$V(r) = V_0 + \frac{1}{2}m\omega^2(r - r_0)^2 \ , \tag{1}$$

   ignoring other terms, where $V_0 < 0$, ω is a characteristic frequency, and m is the reduced mass of the two nuclei. We ignore other terms in the expansion.

(a) Find the energy eigenvalues for bound states of the nuclear motion of this diatomic molecule. When you consider the rotational motion, you can assume that the radial potential is so steep that the radius is fixed at $r = r_0$. Note that the Hamiltonian now breaks into two terms which commute with each other, so they can be diagonalized separately.

(b) Consider an H_2 molecule, so that the two nuclei are identical particles (protons). If the nuclear spins are in a total spin singlet state (that is, total $s = 0$), what values of the orbital angular momentum quantum numbers are allowed? Note that exchanging the two nuclei is equivalent to taking $\vec{x} \to -\vec{x}$ in the spatial wavefunction, which is $r \to r$, $\cos\theta \to -\cos\theta$, $\phi \to \phi + \pi$ in spherical polar coordinates.

(c) Including the nuclear orbital angular momentum and electron spin, are H_2 molecules fermions or bosons?

4. **Postponed**

5. **Postponed**