# PHYS-4601 Homework 10 Due 1 Dec 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Rotations** *parts of Griffiths 4.56*

   (a) Argue that $\exp[i\varphi L_z/\hbar]$ is a rotation around the $z$ axis by showing that

   $$e^{i\varphi L_z/\hbar} \cdot \psi(\phi) = \psi(\phi + \varphi) \tag{1}$$

   for any angular wavefunction $\psi(\phi)$ that can be written as a Taylor series around $\phi$. *Hint*: Use the identification that $L_z = -i\hbar\partial/\partial\phi$ and write both sides as a power series.

   As a result, the angular momentum operators are called the *generators* of rotations. In general, $\hat{n} \cdot \vec{L}/\hbar$ generates rotations around the unit vector $\hat{n}$. Furthermore, the rotations of spinors are generated by the spin angular momentum operators.

   (b) What is the $2 \times 2$ matrix corresponding to a rotation of $2\pi$ around the $z$ axis for spin 1/2? How does it compare to what you expected?

   (c) Construct the $2 \times 2$ matrix corresponding to a rotation of $\pi$ around the $x$ axis for spin 1/2. Show that it takes the $S_z$ eigenstate $|+\rangle$ into $|-\rangle$.

2. **Hadron Spins** *Griffiths 4.35 plus*

   *Quarks* are elementary particles with spin 1/2, which we see in bound states called *hadrons*. Hadrons come in two varieties. In the following, assume that the quarks have zero orbital angular momentum.

   (a) *Mesons* are formed of a quark and antiquark (think of it as two quarks). What are the possible total spin quantum numbers of a meson?

   (b) *Baryons* are formed of three distinct quarks. What are the possible total spin quantum numbers? How many complete sets of states are there for each of those total spins?

3. **Spin Interactions**

   (a) Two spin 1/2 particles are fixed in position but have interacting spins. Their Hamiltonian is

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} \tag{2}$$

   for some constant $J$. Here $S^{(i)}$ is the spin operator of the $i$th particle. Find the energy eigenvalues of this system, their degeneracies, and the corresponding eigenstates. *Hint*: You will want to work in terms of the total spin quantum numbers.

   (b) The two spins have the same gyromagnetic ratio $\gamma$. In the presence of a magnetic field, the Hamiltonian becomes

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} - \gamma\vec{B} \cdot \left(\vec{S}^{(1)} + \vec{S}^{(2)}\right) \ . \tag{3}$$

   Now find the energy eigenvalues and their degeneracies. You may take $\vec{B}$ to lie along the $z$ direction.

   (c) Suppose that initially the first spin is up, while the second spin is down (ie, $|+\rangle$ and $|-\rangle$ respectively). What is the probability of finding the first spin up at a later time $t$? If both spins are initially up, what is the probability?