

Quantum Mechanics II PHYS-4601

First In-Class Test

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Instructions:

- Do not turn over until instructed.
- You will have 75 minutes to complete this test.
- No electronic devices or hardcopy notes are allowed.
- This test has 2 pages of questions (4 total pages including cover sheets).
- **Answer all questions briefly and completely.**
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Formulae:

- Schrödinger Equation
 - time-dependent and position-basis time-independent

$$i\hbar \frac{d}{dt} |\Psi\rangle = H |\Psi\rangle, \quad \langle \vec{x} | H | \psi \rangle = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{x}) + V(\vec{x}) \psi(\vec{x}) = E \psi(\vec{x})$$

- Ehrenfest's Theorem and time-evolution operator

$$\frac{d\langle \mathcal{O} \rangle}{dt} = \frac{i}{\hbar} \langle [H, \mathcal{O}] \rangle + \left\langle \frac{\partial \mathcal{O}}{\partial t} \right\rangle, \quad e^{-iHt/\hbar}$$

- Momentum and Position
 - Inner product of eigenstates (change of basis) in 1D

$$\langle x | p \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \quad \text{or} \quad |p\rangle = \int dx \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} |x\rangle$$

Normalization is given for infinite volume

- Operator commutator $[x, p] = i\hbar$
- Operator representation in position basis

$$\langle x | p | \psi \rangle = -i\hbar \frac{d\psi}{dx} \Leftrightarrow p \simeq -i\hbar \frac{d}{dx}$$

- Uncertainty
 - The uncertainty $\sigma_{\mathcal{O}}$ obeys $\sigma_{\mathcal{O}}^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$
 - Heisenberg Uncertainty Principle $\sigma_A \sigma_B \geq (1/2) |\langle [A, B] \rangle|$

- Delta Functions in Potentials $V(x) = -\alpha\delta(x - a) + \dots$

$$\lim_{\epsilon \rightarrow 0} \left(\frac{d\psi}{dx}(a + \epsilon) - \frac{d\psi}{dx}(a - \epsilon) \right) = -\frac{2m\alpha}{\hbar^2} \psi(a)$$

- Infinite Square Well $V(x) = 0$ for $-a < x < a$, $V(x) = \infty$ otherwise

– Eigenfunctions

$$\psi_n(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{n\pi x}{2a}\right) \quad (n \text{ odd}) \text{ or } \psi_n(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi x}{2a}\right) \quad (n \text{ even})$$

– Energy Eigenvalue $E_n = (\hbar^2/2m)(n\pi/2a)^2$

- Probability current $\vec{j} = (i\hbar/2m) (\Psi \vec{\nabla} \Psi^* - \Psi^* \vec{\nabla} \Psi)$

- 1D Harmonic Oscillator $V(x) = (1/2)m\omega^2 x^2$

– Stationary State Wavefunctions (see Hermite polynomials below)

$$\langle x|n\rangle = \psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right) \exp\left[-\frac{m\omega x^2}{2\hbar}\right]$$

– Ladder operators

$$a = \sqrt{\frac{m\omega}{2\hbar}} x + i \frac{p}{\sqrt{2\hbar m\omega}}, \quad a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad [a, a^\dagger] = 1$$

– Hamiltonian $H = \hbar\omega (a^\dagger a + 1/2)$

- Pure math

– Integrals

$$\int_0^\infty dx x^p e^{-x/b} = p! b^{p+1} \quad (b > 0), \quad \int_{-\infty}^\infty dx e^{-ax^2+bx} = \sqrt{\frac{\pi}{a}} e^{b^2/4a} \quad (\text{Re } a > 0)$$

– Trig and hyperbolic trig functions

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}, \quad \cosh \theta = \frac{e^\theta + e^{-\theta}}{2}, \quad \sinh \theta = \frac{e^\theta - e^{-\theta}}{2}$$

Derivatives, integrals, and angle-addition formulae follow from these

TABLE 2.1: The first few Hermite polynomials, $H_n(\xi)$.

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| $H_0 = 1,$ $H_1 = 2\xi,$ $H_2 = 4\xi^2 - 2,$ $H_3 = 8\xi^3 - 12\xi,$ $H_4 = 16\xi^4 - 48\xi^2 + 12,$ $H_5 = 32\xi^5 - 160\xi^3 + 120\xi.$ |
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