PHYS-3301 Winter Homework 8 Due 8 Mar 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Harmonic Motion Time Delays**

   A rocket flies back and forth around a resting space station with position $x = A\sin(\omega t)$ (t is the time on the space station). The period of motion is $2\pi/\omega$.

   (a) *inspired by Barton 6.4* First, assume $\omega A \ll c$. (This motion is then similar to a nonrelativistic mass on a spring or nonrelativistic pendulum.) Find the proper time on the rocket as a function of space station time t. Chose integration constants so that $\tau = 0$ at $t = 0$. Make appropriate approximations.

   (b) Sketch $\tau(t)$ for one period $0 < t < 2\pi/\omega$ and show for reference the 45-degree line $\tau = t$. Then find the difference between the total elapsed time t and the total elapsed proper time of the rocket over one period (this is the "time delay").

   (c) Find the time delay over one period in the case that $\omega A = c$. As a fraction of the period, is the time delay larger in this case or for $\omega A \ll c$?

2. **Ultrarelativistic Velocity Addition** *from Hogg 4.8*

   A neutral pion particle (π^0) of mass M is produced at rest with respect to the lab frame. In one possible but rare decay, it produces an electron and positron (anti-electron) which move off in opposite directions, each with mass m and relativistic γ factor of $\gamma = M/2m \approx 100$.

   (a) Since γ is so large, the speed of the electron or positron relative to the lab can be written as $u/c = 1-\epsilon$. Find ϵ to the lowest order in the small number m/M (that is, if ϵ is written as a power series in m/M, find the power series out to the lowest power with a nonzero coefficient) and then to 1 significant digit.

   (b) Relative to the electron, what is the positron's speed? Again, write the relative speed as $u/c = 1 - \tilde{\epsilon}$ and find $\tilde{\epsilon}$ to lowest order in m/M and to 1 significant digit.

3. **Gene and George**

   George Lucas and Gene Roddenberry decide to race each other from Earth to Mars over a distance d. Lucas travels in a straight line from Earth to Mars at constant speed, while Roddenberry travels along a semicircle of radius $d/2$ at constant speed. Roddenberry's ship is faster, though, so both arrive at Mars at the same time ($t = T$ in the common rest frame of Earth and Mars). Label Earth's position as $x = -d/2$ and Mars's position as $x = d/2$.

   (a) Who has aged more during the trip? Explain your answer qualitatively.

   (b) Assuming our two racers leave Earth at $t = 0$ in the Earth-Mars frame, write Roddenberry's position and velocity as functions of time t and his proper time τ. Assume that Roddenberry moves in the (x, y) plane.

   (c) What is Roddenberry's velocity in Lucas's rest frame? Write your answer as a function of Roddenberry's proper time. Is this the velocity of circular motion?