PHYS-3301 Winter Homework 6 Due 15 Feb 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Invariance of Light Speed**

   In this question, we'll show that the speed of light is invariant, no matter the direction of the velocity. (Of course, our derivation of the Lorentz transformations showed this for light moving along the relative motion of two frames.) Start in the S frame, where a light beam leaves the origin at time $t = 0$ and reaches point $\vec{x} = (x_0, y_0, 0)$ at time $t = \sqrt{x_0^2 + y_0^2}/c$, hitting a detector there. Now consider a frame S' moving at speed v relative to S along the x axis.

   (a) At what coordinates t', x', y', z' does the light hit the detector? Write your answer in terms of c, x_0, y_0, and v.

   (b) Find the components of the light's velocity in the S' frame by dividing x'/t', etc. What is the speed of light in S'?

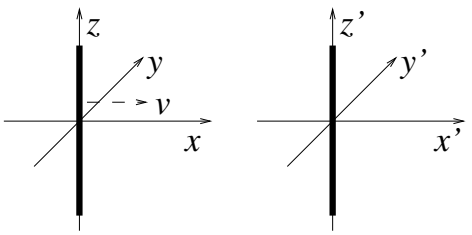
   (c) Why don't we need to think about a component of motion along the z axis to get a general proof of the invariance of the speed of light?

2. **Speed of Gravity**

   In September 2015, the LIGO observatory observed for the first time gravitational waves, which have been identified as coming from two black holes merging at a distance of 1 billion light-years from earth. 0.4 s later, the Fermi Gamma Ray Observatory saw a flash of gamma rays (high-energy light) from the same general direction as the gravitational waves. Assume that the gamma rays and gravitational waves come from the same black hole merger and left at the same time. Calculate $(c_g - c)/c$, where c is the usual speed of light and c_g is the speed of gravitational waves, to one significant figure; you should make valid approximations that allow you to simplify the arithmetic. You may use the conversions that 1 year is 3×10^7 s and that 1 light-year is 9×10^{15} m. *NOTE:* In reality, even if the gamma rays and gravitational waves come from the same event, it's quite possible the gamma rays would have been delayed by dense material around the black hole merger. That means our calculation gives an upper bound for the difference between c_g and c.

3. **The Shape of a Moving Object**

   Frames S and S' are in standard configuration with S' moving at speed v along the $+x$ direction. A cylindrical rod is at rest in S' with its axis along the z' axis, and its cross section forms a circle of radius r centered on the origin in the $x'y'$ plane (see the diagram). Show that the cross section forms an ellipse moving in the x direction in the S frame and find the eccentricity e of the ellipse.



Hint: The equation for an ellipse with axes along the x and y axes and center at (x_0, y_0) is

$$\frac{(x-x_0)^2}{r_x^2} + \frac{(y-y_0)^2}{r_y^2} = 1 \ , \tag{1}$$

and the eccentricity is $e = \sqrt{|r_x^2 - r_y^2|}/\max(r_x, r_y)$. A circle is an ellipse with $e = 0$ ($r_x = r_y$).

4. **Relativistic Monorail** *inspired by Resnick & Halliday and by Hartle*

   A monorail stretches between two halves of a large starbase. There are lights attached to the monorail, evenly spaced with separation L in the rest frame of the base, and the lights flash simultaneously in that frame. In the frame of the train moving along the monorail, the lights are a distance $12L/13$ apart.

   (a) Use length contraction to find the speed of the train with respect to the monorail.

   (b) In the train's frame, how long is it between flashes of neighboring lights? Does the light in front of the train (ie, the one the train is moving toward) flash before or after the light behind the train?

   (c) The rear of the train passes one of the lights just as it flashes. How much time passes before the light reaches the front of the train in the train's frame? In the frame of the starbase? The proper length of the train is $L/3$.