PHYS-3301 Winter Homework 1 Due 11 Jan 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

Helpful Formulae: You might find the integral

$$\int_0^\infty dx\, x^n e^{-x/a} = n!\, a^{n+1} \quad (a > 0,\ n = 0, 1, 2, \cdots) \tag{1}$$

useful. You will also want to know that the angular wavefunctions satisfy

$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, \sin\theta\, |Y_\ell^m(\theta,\phi)|^2 = 1\ . \tag{2}$$

1. **Particle in a Thick Spherical Shell**

   Consider a central potential

   $$V(r) = \begin{cases} \infty & 0 < r < a\ ,\ 2a < r \\ 0 & a < r < 2a \end{cases} \tag{3}$$

   in three dimensions. Consider the $l = 0$ states only. *Hint:* Remember that this potential means that the particle moves freely for $a < r < 2a$ and that the wavefunction goes to zero at $r = a$ and $2a$.

   (a) Find the energy eigenfunctions and eigenvalues. You do not have to normalize the eigenfunctions.

   (b) (5 points) Consider the 2nd excited state of this system. Sketch the wavefunction ψ as a function of radius for $a/2 < r < 5a/2$.

2. **Ground State Radii of Hydrogen**

   For this problem, consider the ground state of hydrogen as described by the wavefunction given in table 5.1 of French & Taylor (with $Z = 1$). Give your answers in terms of the Bohr radius.

   (a) Find expectation value of the radius $\langle r\rangle$ for this state.

   (b) Find the root-mean-square (rms) radius $r_{rms} \equiv \sqrt{\langle r^2\rangle}$ for this state.

   (c) Find the most probable radius $\bar{r}$ for this state. $\bar{r}$ is defined as the value such that

   $$\lim_{\Delta\to 0}\left(\frac{1}{\Delta}\right)\int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, \sin\theta \int_{\bar{r}}^{\bar{r}+\Delta} dr\, r^2 |\psi(r,\theta,\phi)|^2 \tag{4}$$

   is maximized.

3. **High Angular Momentum States**

   In this question, consider an electron in a state of hydrogen with the largest possible value of angular momentum $\ell = n - 1$. From our class notes, we know that the radial wavefunction can be written as

   $$R(r) = Ar^{n-1}e^{-r/na_0}\ . \tag{5}$$

   (a) Find the normalization constant A.

   (b) Find $\langle r^2\rangle$ in this state.