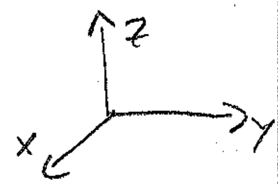
Newtonian + Statistical Mechanics

Galilean Relativity

- Every time we solve a problem or make a measurement, we have chosen a <u>reference frame</u>
  - This is a set of coordinates centered around some point or observer
    - Includes x, y, z axes
    - Time coordinate (or axis) t



  - How can we change (or transform) reference frames from frame S to S'?
    - Time translation $t' = t + b$
    - (Spatial) translation $\vec{x}' = \vec{x} + \vec{b}$
    - Rotations $\vec{x}' = R\vec{x}$, R = matrix
    - Boosts $\vec{x}' = \vec{x} + \vec{v}t$ (change of velocity)

    More details later

  - Is physics the same in all reference frames?
    - If the transformation is uniform (ie, $b, \vec{b}, R, \vec{v}$ = constants) then yes.
      + Specifically, consider a free object (no external force).
      + Then $\vec{a} = 0$ in one frame means $\vec{a} = 0$ in others.
    - But suppose the transformation is not uniform
      + S' accelerated w.r.t S: $\vec{x}' = \vec{x} + \frac{1}{2}\vec{a}_0 t^2$
      + Then a free object in S ($\vec{a} = 0$) has acceleration
        $$\vec{a}' = d^2\vec{x}'/dt^2 = \vec{a}_0 \neq 0 \quad \text{in } S'$$
    - If a free object does not accelerate, you are in an <u>inertial frame</u>.

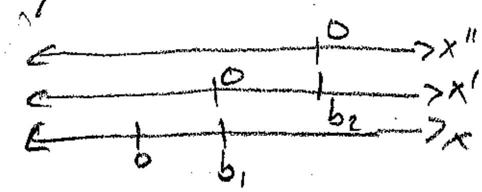
- The <u>Relativity Principle</u>
  + All laws of nature have the same form in all inertial frames
  <u>or</u> <u>equivalently</u>

+ No experiment performed in one inertial frame can determine its motion relative to another inertial frame (or orientation or origin)

- The uniform translations, rotations, + boosts go from one inertial frame to another. Let's look at these again

• Translations: Consider 1 space dimension. It's easy to generalize.

Start with coordinates x and shift the origin of x' a distance b_1 to the right.



Then, for any point, $x' = x - b_1$.

To "add" translations, note $x'' = x' - b_2 = (x - b_1) - b_2 = x - (b_1 + b_2)$

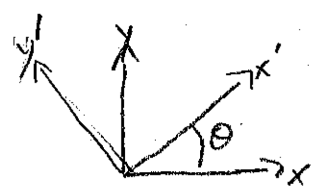
• Rotations:

+ We often define 3D positions as vectors $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and say physically that a rotation takes $\vec{x}' = R\vec{x}$ (matrix multiplication)

This is sometimes taken as the definition of a vector in physics.

+ For ex, a rotation around z by θ is

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(leaves z coordinate alone)

+ We will use index notation $x^i = (x^1 = x, x^2 = y, x^3 = z)$ with rotation

$$x'^{i'} = \sum_j R^{i'}{}_j \, x^j \qquad \left\{ \begin{array}{l} 1^{st} \text{ index} = \text{row } \# \\ 2^{nd} \text{ index} = \text{column } \# \end{array} \right\} \text{ work out matrix multiplication}$$

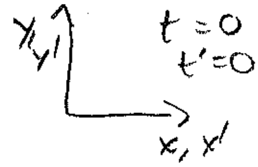
We also use Einstein summation convention that a repeated index is summed over

$$R^{i'}{}_j \, x^j \equiv \sum_{j=1}^{3} R^{i'}{}_j \, x^j$$

You will practice matrix operations in index notation on HW.

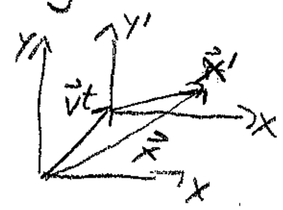
• <u>Boosts</u>: Uniform changes of velocity.

+ Consider 2 inertial frames with S' moving at velocity $\vec{v}$ w.r.t. S.
  a) So the spatial origin $\vec{O}'$ moves w/velocity $\vec{v}$ w.r.t $\vec{O}$
  b) Use translations to set time/space origins equal. That is,
     $(t, \vec{x}) = (0, \vec{0})$ coincides with $(t', \vec{x}') = (0', \vec{O}')$
  c) use rotations to line up axes.



+ All clocks in all frames are synchronized $t = t'$
  You must have infinite speed signalling to do that.

+ Vector addition gives
$$\vec{x}' = \vec{x} - \vec{v}t$$



+ The inverse transformation is from interchanging primed w/unprimed and taking $\vec{v} \to -\vec{v}$.

+ "Standard configuration" in the book means $\vec{v}$ along x-axis
$$t' = t, \quad x' = x - vt, \quad y' = y, \quad z' = z.$$

– Active vs Passive Transformations: What do these transformations represent?
  • We might think of looking at the same process from different frames, for example, choosing your time origin appropriately.
    These are <u>passive transformations</u>
  • Or you could think of a single frame but 2 related processes,
    Such as particles moving along x or y (related by rotation).
    These are <u>active transformations</u>.
  • There's not a whole lot of difference, so they are usually used interchangeably.

– We're talking about <u>kinematics</u> (description of motion)
  Look at other kinematic quantities
  • <u>Velocity</u>: $\vec{u} = d\vec{x}/dt$ + Unchanged under translations
    + Is a vector under rotations } why?

+ Imagine a boost by $\vec{v}$ ($\vec{u}$ = particle velocity, $\vec{v}$ = velocity between frames)

Then $\vec{u}' = \frac{d\vec{x}'}{dt} = \frac{d(\vec{x}-\vec{v}t)}{dt} = \vec{u} - \vec{v}$ (a passive transformation)

Notice: we can also ask how to "add" velocities for the sequence $S \xrightarrow[v_1]{} S' \xrightarrow[v_2]{} S''$ in std. config. Then $S \xrightarrow[v]{} S''$ with $v = v_1 + v_2$. Derive.

If we think of particles in frame S w/ velocities $0, v_1, v$, these are related by active boosts.

- Can you work out other quantities? Like acceleration $\vec{a}' = \vec{a}$ under boosts

- Using the relativity principle in <u>dynamics</u> (laws of motion, like Newton's)

  - <u>Covariance</u>: All true laws have the same form in all inertial frames.

    That is, they are <u>co-variant</u> ← the lhs + rhs (all terms) transform the same way. (Bock calls this form-invariant)

  - Means we can use whatever frame is easiest to solve a problem, then convert back to the frame given in the problem. The <u>center-of-mass frame</u> aka <u>center-of-momentum frame</u> aka <u>CM frame</u> is very useful.

    Definition of
  - CM frame: total momentum $\sum_i \vec{p}_i = 0$ of all particles.

    + Means that the center of mass $\vec{R} \equiv \sum_i m_i \vec{x}_i / \sum_i m_i$ is constant (prove).

      + <u>Example</u> 2 colliding particles. Initially $\vec{p}_1 = -\vec{p}_2$. Finally, $\vec{p}_1' = -\vec{p}_2'$. (Conservation)

        For an elastic collision, KE conserved.

$$K = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} \Rightarrow p_1^2\left(\frac{1}{2m_1}+\frac{1}{2m_2}\right) = p_1'^2\left(\frac{1}{2m_1}+\frac{1}{2m_2}\right)$$

$\Rightarrow$ Each particle has the same <u>speed</u> before + after collision.

  - You can split KE into KE of CM plus KE "internal" to CM frame.

    2 particle example → + If CM has velocity $\vec{V}$, velocity of particle in CM frame is $\vec{u} - \vec{V}$.

    + Then KE is

$$K = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1(\vec{u}_1^{cm} + \vec{V})^2 + \frac{1}{2}m_2(\vec{u}_2^{cm} + \vec{V})^2$$

$$= \frac{1}{2}m_1 u_1^{cm\,2} + \frac{1}{2}m_2 u_2^{cm\,2} + \frac{1}{2}(m_1+m_2)V^2 + (m_1\vec{u}_1^{cm} + m_2\vec{u}_2^{cm})\cdot\vec{V}$$

This is

$$K = \underset{\text{CM frame KE}}{K_{int}} + \underset{\text{KE of CM}}{\tfrac{1}{2}MV^2} + \vec{0}\cdot\vec{v}$$

by definition of CM

+ There is one more simplification. Let the <u>relative</u> <u>velocity</u> of 1 + 2 be $\vec{u} = \vec{u}_1 - \vec{u}_2 = \vec{u}_1^{cm} - \vec{u}_2^{cm}$. But $\vec{u}_2^{cm} = -\frac{m_1}{m_2}\vec{u}_1^{cm} \Rightarrow \vec{u} = \vec{u}_1^{cm}\left(1+\frac{m_1}{m_2}\right)$

$$K_{int} = \tfrac{1}{2}m_1 (u_1^{cm})^2 + \tfrac{1}{2}\frac{m_1^2}{m_2}{u_1^{cm}}^2 = \tfrac{1}{2}m_1\left(1+\frac{m_1}{m_2}\right)\left(1+\frac{m_1}{m_2}\right)^{-2}u^2 = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}u^2.$$

We say $K_{int} = \frac{1}{2}\mu u^2$ where $\mu = \frac{m_1 m_2}{m_1+m_2}$ is the <u>reduced</u> <u>mass</u>.

+ Example: 2 lumps of clay of masses m_1, m_2 and velocities u_1, u_2 collide in 1D. They stick together + heat up. How much heat is released?
Heat is internal CM energy (indeed, the clay is motionless in CM frame after)
So the released heat is $\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1-u_2)^2$

- Another Example: Suppose a bird is sitting on the ground in a wind of speed v.
How high can the bird fly without flapping (expending energy)?
In wind's frame, bird (+ ground) has speed v. The bird can clearly rise to height $gy' = v^2/2$. But distance $y' = y$ is frame-invt.

– Let's look now at the transformation laws of some dynamical quantities
  - Mass: inherent property, so invariant under translations, rotations, boosts.
    Same for total mass, $M \equiv \sum_i m_i$
  - Momentum: $\vec{p} = m\vec{u}$. Invt under translations, vector under rotations, and $\vec{p}' = \vec{p} - m\vec{v}$ under boosts. Total momentum is similar $\vec{P}' = \vec{P} - M\vec{v}$.
  - Potential energy: A central potential (depends only on separation) is invt. $V' = V$
  - Kinetic energy: $k = p^2/2m \Rightarrow k' = k - \vec{p}\cdot\vec{v} + \frac{1}{2}mv^2$
    We have seen the total KE: $K = P^2/2M + K_{int} \Rightarrow K' = K - \vec{P}\cdot\vec{v} + \frac{1}{2}Mv^2$
  - Physical laws must be covariant:
    + Momentum conservation $\Delta\vec{P} = 0$. In another frame, $\Delta\vec{P}' = \Delta(\vec{P} - M\vec{v}) = \Delta\vec{P} = 0$ b/c M is constant as is $\vec{v}$.
    + Can you show the same for energy conservation?