PHYS-2106 Winter Homework 8 Due 13 Mar 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

Hint: You will want to remind yourself about integration by parts and trigonometric identities (like angle addition formulas, etc) for this assignment.

1. **Simple Fourier Series**

   Fourier series are unique, so you can find Fourier series without using integrals to compute the Fourier coefficients in some cases. Find the following Fourier series; these all have a finite number of terms.

   (a) The cosine series for $f(x) = \cos^2(x)$ as a function of period 2π.

   (b) The complex exponential series for $f(x) = \cos(4x) - \sin^3(x)$ as a function of period 2π.

2. **Sums from Parseval's Theorem** *inspired by Boas & Spiegel*

   Parseval's theorem is another way of determining an infinite sum by using a Fourier series. In this problem, you will first calculate the requested *complex exponential* Fourier series and then use Parseval's theorem to compute the requested sums.

   (a) Find the complex Fourier series for the *full-wave rectifier* function $f(x) = |\sin(x)|$ for $-\pi < x < \pi$.

   (b) Use Parseval's theorem and your previous result to calculate $\sum_{k=1}^{\infty}(2k+1)^{-2}(2k-1)^{-2}$.

   (c) Find the complex Fourier series for the function $f(x) = x^2$ on the interval $-1 < x < 1$.

   (d) Use Parseval's theorem and your previous result to find $\sum_{n=1}^{\infty} n^{-4}$.

3. **Finite Wavetrain**

   Find the Fourier transform of the function $f(x) = e^{ik_0 x}\Theta(N\pi/k_0 - |x|)$, where $\Theta(y)$ is the Heaviside step function. Assuming that N is an integer (so that the wavetrain consists of N periods of the plane wave), for what values of k does $\tilde{f}(k) = 0$?

4. **Practice Problems (NOT GRADED)**

   For your own practice, the following problems from the course textbooks are useful (and mostly have solutions in the texts):

   Spiegel problems 7.26-31, 8.15-16, 9.28-31,39-41,43

   Unfortunately, I have not found any solved problems that we have not already done regarding Gaussian integrals or change of basis in linear algebra. I advise reviewing your notes and homework.