PHYS-2106 Winter Homework 4 Due 6 Feb 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Inverses and Systems of Equations**

   (a) Carry out the *LU* decomposition (as defined in class and the reading) of the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and then use that decomposition to verify that $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

   (b) *inspired Kreyszig* Solve the system of equations $Ax = b$ for

   $$A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix} . \qquad (1)$$

2. **Pauli Matrices Again**

   Recall from our previous assignment that the Pauli matrices are defined by

   $$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} , \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \qquad (2)$$

   (a) Show that the Pauli matrices are Hermitian.

   (b) Show that the Pauli matrices are unitary.

   (c) Find the eigenvalues and eigenvectors for all three Pauli matrices. Normalize the eigenvectors so that their norms are equal to one.

3. **Eigenvectors of a Degenerate Eigenvalue**

   Suppose a matrix A has two linearly independent eigenvectors x and y with the same eigenvalue λ. Show that any linear combination $ax + by$ (with a, b any scalars) is also an eigenvector of A with eigenvalue λ.

4. **Moment of Inertia Tensor** *partly from Boas, partly from Marion & Thornton*

   The angular velocity $\vec{\omega}$ and angular momentum $\vec{L}$ of a rotating object are generally related by the linear equation $\vec{L} = I\vec{\omega}$, where I is a 3×3 matrix called the *moment of inertia tensor* of the object.

   (a) The eigenvectors of I are called the principal axes, and the corresponding eigenvalues I_1, I_2, I_3 are the principal moments of inertia. Argue that $\vec{L}$ and $\vec{\omega}$ are parallel if and only if $\vec{\omega}$ lies along a principal axis.

   Suppose a cube of side b and uniform density with total mass M is aligned along the x, y, z axes with one corner at the origin. Its moment of inertia tensor is

   $$I = Mb^2 \begin{bmatrix} \frac{2}{3} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{2}{3} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{2}{3} \end{bmatrix} . \qquad (3)$$

(b) If the cube is rotating around the x axis with angular velocity $\vec{\omega} = \begin{bmatrix} \omega & 0 & 0 \end{bmatrix}^T$, find the angular momentum.

(c) Find the principal moments of the cube. You may use Maple to calculate any determinants or solve the characteristic equation (using the `solve` command), but you must attach a printout of your Maple worksheet in that case.

5. **Practice Problems (NOT GRADED)**

For your own practice, the following problems from the course textbooks are useful (and mostly have solutions in the texts):

- RHB chapter 8 problems 1,3,7(a,b,c),17,33
- Spiegel chapter 15 supplementary problems 49,63,76,82,83,89,90,96(a,b)