PHYS-2106 Winter Homework 10 Due 27 Mar 2017

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Separable Variables Equations** *RHB 14.2, Boas, and others*

   Solve the following differential equations for $y(x)$ by separating variables. Fix any integration constant by choosing your solution to satisfy the given initial conditions. A prime denotes the derivative with respect to x.

   (a) $y' - xy^3 = 0$ for $y(0) = 1/2$
   (b) $x^2y' + xy^2 = 4y^2$ for $y(1) = 1/5$
   (c) $xy' = y$ for $y(2) = 3$.
   (d) $y' - xy = x$ for $y(0) = 1$
   (e) $y' = ry(1-y)$ for $y(0) = y_0$ (this is the *logistic equation* often used to describe population growth or the learning curve)

2. **Bernoulli's Equation** *Arfken-Weber*

   Bernoulli's equation is written as

   $$\frac{dy}{dx} + f(x)y = g(x)y^n \ , \tag{1}$$

   which is nonlinear and generally neither exact nor separable. Show that the function $z(x) = y(x)^{1-n}$ satisfies a linear equation (which we know how to solve), however.

3. **RC Circuit** *RHB 14.8 extended*

   Kirchoff's laws for a circuit with resistance R and capacitance C gives a linear differential equation $R(dQ/dt) + Q/C = V(t)$ for the charge on the capacitor, where $V(t)$ is the potential across the circuit.

   (a) Find the solution to the homogeneous equation with $V(t) = 0$.
   (b) We will find the particular solution for $V(t) = e^{i\omega t}$ by two methods. First, use the general solution method discussed in the class notes.
   (c) As an alternative method, guess $Q(t) = Be^{i\omega t}$ and solve the differential equation for the constant B. Verify that this matches the solution from the previous part.
   (d) Argue that the particular solution for $V(t) = e^{-i\omega t}$ is the complex conjugate of the solution from parts (b,c). Find this solution. Then argue that the particular solution for $V(t) = \cos(\omega t)$ is 1/2 times the sum of the two conjugate solutions. Find it.
   (e) Finally, use the solution to the homogenous equation to find the solution for $V(t) = \cos(\omega t)$ such that $Q(0) = 0$.

4. **Exact Equations** *from Boas*

   A prime indicates a derivative with respect to x.

   (a) Show that the differential equation $(x-y)y' = -(1+x+y)$ is exact and find the general solution.
   (b) Some equations are not exact as written but can be made exact by multiplying all terms by a function called the *integrating factor*. Show that the equation $3xy^2y' + 3y^3 = 1$ becomes exact when it is multiplied by the integrating factor x^2 and find the general solution.