Mathematical Physics II PHYS-2106
First In-Class Test

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Instructions:

- Do not turn over until instructed.
- You will have 50 minutes to complete this test.
- No electronic devices or hardcopy notes are allowed.
- INSTRUCTIONS REGARDING TEST LENGTH WILL GO HERE.
- **Answer all questions briefly and completely.**
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Formulae

- $\delta_{ij} = 1$ for $i = j$, $= 0$ for $i \neq j$; $\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = -\epsilon_{213} = -\epsilon_{132} = -\epsilon_{321} = 1$, other components zero

- Matrix relations in terms of elements in some basis

  - $(A^T)_{ij} = A_{ji}$, $(A^\dagger)_{ij} = A^*_{ji}$
  - $(A + B)_{ij} = A_{ij} + B_{ij}$, $(AB)_{ij} = A_{ik}B_{kj}$
  - Inner product of column vectors $x^\dagger G y = x_i^* G_{ij} y_j$; for orthonormal basis $x^\dagger y = x_i^* y_i$.

- Trace: defined as $\text{tr}(A) = A_{ii}$; linear $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$; cyclic $\text{tr}(AB) = \text{tr}(BA)$

- Determinant:

  - = sum of element times cofactor along any row or down any column
  - $\det(A^T) = \det(A)$, $\det(A^\dagger) = \det(A^*) = \det(A)^*$, $\det(\lambda A) = \lambda^N \det(A)$, $\det(AB) = \det(A)\det(B)$
  - Changes sign if you reverse two rows or columns
  - Unchanged if you add a multiple of one row (column) to another row (column)

- General formula for inverse: $A^{-1} = \text{cof}(A)^T / \det(A)$; $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- Eigenvalues/eigenvectors: characteristic equation $\det(\lambda 1 - A) = 0$

- Types of matrices

  - (anti)symmetric $A^T = \pm A$, orthogonal $A^T = A^{-1}$
  - (anti-)Hermitian $A^\dagger = \pm A$, unitary $A^\dagger = A^{-1}$

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