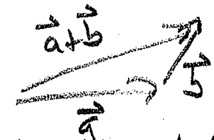
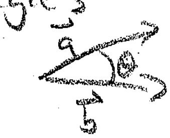
Linear Algebra

Review of Vector Algebra

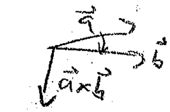
- Physics intuition is that vectors are quantities with magnitude and direction.
  - Define addition/subtraction vectorially



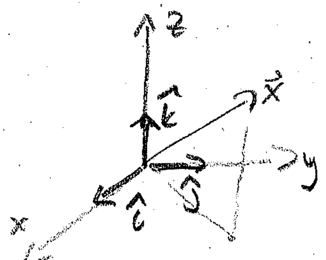
  - Dot product (aka scalar product, inner product) defined in terms of angles



$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta, \qquad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

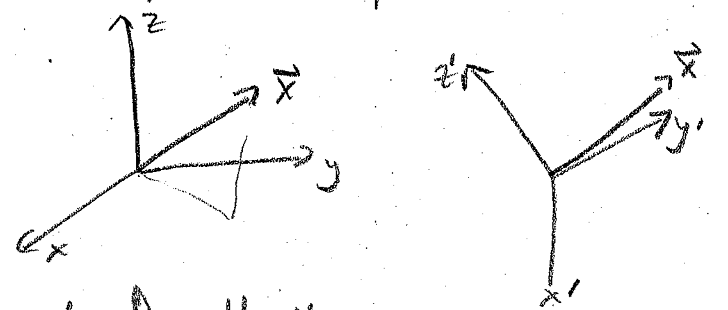


- We can choose axes and unit vectors along the axes



  - Then we can write a vector in terms of components
    + $\vec{x} \equiv x\hat{i} + y\hat{j} + z\hat{k} \equiv x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$
    + Alternately, we can represent the vector as a column matrix $\vec{x} \simeq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ or just by the components x_i
  - We can reformulate vector products in terms of components
    + $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = \sum_i a_ib_i \equiv a_ib_i$
      a) Einstein summation convention means sum over repeated indices (that is, drop the summation symbol)
      ^over all values
      b) With the Kronecker delta symbol, $\delta_{ij} = \begin{cases} 1 & \text{for } i=j \\ 0 & \text{for } i\neq j \end{cases}$,
      we can write $\vec{a}\cdot\vec{b} = a_i\delta_{ij}b_j$.
    + Cross products are $(\vec{a}\times\vec{b})_i = \epsilon_{ijk}a_jb_k$
      where the Levi-Civita (or totally antisymmetric) symbol is
      $$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = -\epsilon_{213} = -\epsilon_{132} = -\epsilon_{321} = 1, \quad \epsilon_{ijk} = 0 \text{ otherwise}$$
    + These can simplify derivations of some vector identities

- The vector is a mathematical object independent of axes, even if the components depend on axes:



$x_i \neq x_{i'}$

→ We will extend all these concepts shortly