PHYS-4601 Homework 8 Due 5 Nov 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Matrix Elements for the Harmonic Oscillator**

   Calculate the matrix elements $\langle n|x|m\rangle$ and $\langle n|p^2|m\rangle$ for $|n\rangle, |n'\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals.

2. **Orthonormality of Harmonic Oscillator Eigenstates**

   Consider the eigenstates of the harmonic oscillator. *Hint:* This problem will be easier if you use techniques and results from previous homework assignments. You may also find the Gaussian integral formulas on the back cover of the text useful.

   (a) The Hermite polynomials satisfy the *Rodrigues formula*

   $$H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n} e^{-\xi^2} \ . \tag{1}$$

   Use this to show that H_n is an nth order polynomial of the form $H_n(\xi) = 2^n \xi^n + \cdots$. Then show that the eigenstate wavefunctions

   $$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2} \quad (\xi = \sqrt{\frac{m\omega}{\hbar}}\, x) \tag{2}$$

   are orthonormal (with the usual L^2 inner product).

   (b) Now define the energy eigenstates using the raising operator, $|n\rangle = (a^\dagger)^n|0\rangle/\sqrt{n!}$. First show that $[a, (a^\dagger)^n] = n(a^\dagger)^{n-1}$. Then show using operator techniques that these states are orthonormal (ie, $\langle n'|n\rangle = \delta_{n,n'}$).

3. **Coherent States** *based on Griffiths 3.35*

   In this problem, we will study *coherent states*, which are eigenfunctions of the lowering operator

   $$a|\alpha\rangle = \alpha|\alpha\rangle \ , \tag{3}$$

   where the eigenvalue α is generally complex.

   (a) Is there any energy eigenstate that is a coherent state? If so, list which energy eigenstate(s) are coherent and give the eigenvalue(s).

   (b) Show that a coherent state can be written as

   $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \ . \tag{4}$$

   To do that, you will first want to show that $\alpha\langle n|\alpha\rangle = \sqrt{n+1}\langle n+1|\alpha\rangle$. That gives you a recursion relation that the series (4) satisfies. Then you can check that $|\alpha\rangle$ is normalized (remember that $|n\rangle$ are orthonormal).

   (c) If $|\alpha\rangle$ is the initial state of the system, show that the state at time t is still a coherent state with eigenvalue $\alpha(t) = \alpha e^{-i\omega t}$.

   (d) Find the expectation values of x and p in the coherent state $|\alpha\rangle$ (use the ladder operators). Argue that a coherent state behaves like a classical harmonic oscillator by combining your result with the time evolution given above.