PHYS-4601 Homework 3 Due 1 Oct 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Parity Operator**

   Define the operator (in 1 dimension)

   $$P = \int_{-\infty}^{\infty} dx\, |-x\rangle\langle x| \ . \tag{1}$$

   This is called the *parity operator* because it reflects the spatial axis.

   (a) Is P Hermitian? Prove your answer.

   (b) Prove that $P^2 = 1$ (the identity operator).

2. **Expectation and Uncertainty**

   Consider an observable L with three eigenvalues +1, 0, and -1 and corresponding eigenstates $|+1\rangle, |0\rangle, |-1\rangle$. We have a system in state

   $$|\psi\rangle = \frac{1}{3}\Big(|+1\rangle + 2e^{i\beta}|0\rangle + 2|-1\rangle\Big) \ . \tag{2}$$

   (a) What is the probability of measuring each of the three eigenvalues of L?

   (b) Find the expectation value and uncertainty of a measurement of L.

   (c) Another observable A acts on the L eigenbasis as

   $$A|+1\rangle = \frac{1}{\sqrt{2}}|0\rangle \ , \quad A|0\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + |-1\rangle\right) \ , \quad A|-1\rangle = \frac{1}{\sqrt{2}}|0\rangle \ . \tag{3}$$

   Find the expectation value and uncertainty of A in state $|\psi\rangle$.

   (d) Finally, show that the uncertainties of L and A satisfy the uncertainty principle in this state.

3. **Some Commutator Relations**

   For operators A, B, C:

   (a) show that

   $$[A, BC] = [A, B]C + B[A, C] \ . \tag{4}$$

   (b) prove the *Jacobi identity*

   $$[[A, B], C] + [[B, C], A] + [[C, A], B] = 0 \ . \tag{5}$$

   (c) prove by induction that

   $$[A, B^n] = n[A, B]B^{n-1} \ , \tag{6}$$

   if $[A, B]$ commutes with B.

   Finally, consider the position and momentum operators, which have $[x, p] = i\hbar$.

   (d) Show using (6) that $[p, f(x)] = -i\hbar df/dx$. Assume $f(x)$ can be written as a Taylor series.