PHYS-4601 Homework 19 Due 17 Mar 2016

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Stark Effect** *based on Griffiths 6.36*

   The presence of an external electric field $E_0\hat{z}$ shifts the energy levels of a hydrogen atom, which is called the Stark effect. Consider the hydrogen atom to be described by the Coulomb potential; the external electric field introduces a perturbation

   $$H_1 = eE_0 z = eE_0 r\cos\theta \ . \tag{1}$$

   We have already seen on homework that the expectation value of this Hamiltonian in the ground state $n = 1$ vanishes, so there is no shift in the ground state energy. In this problem, we consider the degenerate perturbation theory of the $n = 2$ states. *As spin does not enter, do not consider it in this problem.*

   (a) The four states $|2,0,0\rangle$, $|2,1,0\rangle$, and $|2,1,\pm1\rangle$ are degenerate at 0th order. Label these states sequentially as $i = 1,2,3,4$. Show that the matrix elements $W_{ij} = \langle i|H_1|j\rangle$ form the matrix

   $$W = -3aeE_0 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \tag{2}$$

   where empty elements are zero and a is the Bohr radius. *Hint*: Note that L_z commutes with H_1, so only states with the same quantum number m can have nonzero matrix elements; this will save you quite a bit of work. Then use the angular wavefunctions to see that all the diagonal elements of W must vanish. Finally, use the explicit wavefunctions to evaluate the remaining matrix elements of W (there should only be one independent one left).

   (b) Diagonalize this matrix to show that $|\pm\rangle = (1/\sqrt{2})(|2,0,0\rangle \pm |2,1,0\rangle)$ are eigenstates of W. Find the first order shift in energies of $|\pm\rangle$. *Hint*: Note that the corrected eigenstates may still have contributions from other values of the principal quantum numbers n, but that doesn't quite matter.

   (c) Finally, show that the states $|\pm\rangle$ have a nonzero dipole moment $p_z = -e\langle z\rangle$ and calculate it. You should not need to do any more calculations; just use your answer from part (b).

2. **Matrix Perturbation Theory**

   Consider the matrix Hamiltonian

   $$H \simeq \begin{bmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{bmatrix} \tag{3}$$

   with $E_1 \neq E_2$.

   (a) To first order in perturbation theory, find the energy eigenvalues and eigenstates.

   (b) Find the energy eigenvalues to second order in perturbation theory.

   (c) Find the energy eigenvalues and eigenstates exactly. Then expand them as a power series in ϵ and compare to your perturbative answers from parts (a,b).

3. **Sharp Kick**

Consider a particle initially in the ground state of a 1D infinite square well with potential

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases} . \tag{4}$$

At time $t = 0$, the particle receives a kick in the form of a time-dependent potential $\alpha \cos(\pi x/a)\delta(t)$ for small α. What is the probability that the particle is in the first excited state after $t = 0$?