PHYS-4601 Homework 15 Due Date TBA

This homework is due in the dropbox outside 2L26 by a time to be announced. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Physical Representation of Quantum Gates**
   (a) Consider a qbit represented by a charged spin-1/2 particle (such as an electron or proton) so that the bit $|0\rangle$ is spin up and $|1\rangle$ is spin down. Write the Hadamard gate operator as a matrix in the usual basis.
   (b) Then show that this operator is the time evolution operator for the charged particle first exposed to the z component of a magnetic field for an appropriate length of time and then exposed to the y component for the right length of time. You may find a physically irrelevant overall phase.
   (c) Consider the same physical representation of a qbit and show that the quantum NOT gate can be implemented by the time evolution operator of the particle in the x component of a magnetic field for an appropriate length of time (up to an unphysical overall phase).

2. **2-Qbit Gates**
   Consider a 2 qbit system. Choose a basis for the 2 qbit Hilbert space and use it for all parts of this problem.
   (a) Write the CNOT gate operator as a matrix in that basis and show that it is unitary.
   (b) Consider the 1 qbit gate NOT acting only on the first qbit of our two. Write this gate (call it NOT_1) as a matrix in your 2-qbit basis.
   (c) *inspired by Blümel 7.5.4* We can create a new quantum gate G by first acting with the NOT_1 and then CNOT. Give an example of an input 2-qbit state that can be factorized (that is, written as $|\psi\rangle_1|\phi\rangle_2$ for some 1-qbit states $|\psi\rangle$, $|\phi\rangle$) that is turned into an entangled state by G ($G(|\psi\rangle_1|\phi\rangle_2)$ cannot be factorized).

3. **Cloning Means FTL Communication** *based on a problem by Wilde*
   Suppose that Alice and Bob are at two ends of an EPR/Bell experiment. In other words, they are at rest with respect to each other and separated by 5 lightyears, and each receives one of a pair of entangled electrons with total spin state $s = 0$ simultaneously (in their common rest frame). By prior agreement, Alice measures either the S_z or S_x spin of her electron as soon as she receives it, but Bob does not know which spin she measures.

   After Alice's measurement (in their rest frame time), Bob's electron is in some state $|\psi\rangle_B$. Suppose, in contradiction to the no-cloning theorem, Bob can clone his electron's state onto a large number N of other electrons. (For example, Bob can do some quantum operation that takes his $N+1$ electrons from state $|\psi\rangle_B|\uparrow\rangle_1 \cdots |\uparrow\rangle_N$ to state $|\psi\rangle_B|\psi\rangle_1 \cdots |\psi\rangle_N$.) What measurement(s) can Bob do on his extra N electrons that will tell him with great certainty whether Alice measured the S_z or S_x spin of her electron? Explain your answer. (Note that Bob can accomplish his measurement before Alice can tell him her measurement choice, so they can establish faster-than-light communication in this way. This is a good reason for the no-cloning theorem!)