# PHYS-4601 Homework 13 Due 28 Jan 2016

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **States of Baryons**

   *Baryons* are particles made up of three *quarks*, which are themselves spin-1/2 particles. In addition to spin, these quarks possess quantum "numbers" called *flavor* and *color*. A single quark's color has three basis states, red ($|R\rangle$), green ($|G\rangle$), and blue ($|B\rangle$), and the quarks in light baryons have three possible flavors, up ($|u\rangle$), down ($|d\rangle$), and strange ($|s\rangle$). Assume that the ground state for any baryon has a wavefunction that is symmetric under exchange of any of the quarks.

   (a) The $\Delta^{++}$ baryon has spin-3/2, and its charge (+2) means it is composed of 3 up quarks. Therefore, the $m_s = +3/2$ state has flavor-spin state $|u\rangle_1|u\rangle_2|u\rangle_3|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$. Write a possible color state for the three quarks.

   (b) Due to the strong nuclear force, all baryons actually must have the same color state (your answer to part (a)). Now consider the $\Sigma^{*-}$ particle, which is also spin-3/2 and is composed of two down quarks and one strange quark. The $m_s = +3/2$ state has spin state $|\uparrow\rangle_1|\uparrow\rangle_3|\uparrow\rangle_3$; write the flavor state of this baryon.

2. **Carbon As A Multi-electron Atom**

   A carbon atom in a ground state has 4 electrons in $\ell = 0$ orbitals and 2 electrons in $\ell = 1$ orbitals. That is, its electron configuration is $(1s)^2(2s)^2(2p)^2$.

   (a) Start by ignoring the electron Coulomb repulsion, so the energies are given entirely by the hydrogen-like shell energies. By counting the ways to arrange the $p$-orbital electrons, find the ground state degeneracy of carbon in this approximation.

   (b) *inspired by Griffiths 5.13(b)* Now include the electron Coulomb repulsion. Since the $s$-orbital electrons contribute no orbital angular momentum, the total orbital angular momentum is determined by addition of the angular momenta of the two $p$-orbital electrons. Use exchange force arguments to explain why the total electron spin and orbital angular momentum are given by $S = 1, L = 1$ for the ground state of carbon. What is the degeneracy of the ground state using this additional information? *Hint:* You may be interested in knowing that the total $\ell = 2$ states from addition of two $\ell = 1$ states are symmetric while the total $\ell = 1$ states are antisymmetric. These arguments are the logic for one of Hund's rules.

   (c) Hund's third rule requires the total electron configuration for carbon to be ${}^3P_0$. With this last piece of information, what is the actual degeneracy of the carbon atom?

3. **The GHZM Experiment**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture at `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF` . This is the video listed on the reading assignment.

   Three electrons are prepared in the so-called "GHZM" spin state $|\psi\rangle = (|+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3)/\sqrt{2}$ and distributed so that laboratories at locations $A$, $B$, and $C$ each receive one electron.

(a) Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.

(b) If the electrons' total state is written as $|\phi\rangle \otimes |\psi\rangle$, where $|\phi\rangle$ is the spatial state of the electrons, what is $|\phi\rangle$? Each single-electron spatial state is $|A\rangle$, $|B\rangle$, or $|C\rangle$.