PHYS-4601 Homework 12 Due 14 Jan 2016

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Angular Momentum in Hydrogen** *related to Griffiths 4.35*

   Consider an electron in a hydrogen atom with $\ell = 1$ orbital angular momentum (and of course spin $s = 1/2$).

   (a) What are the allowed total angular momentum quantum numbers j for the electron?

   (b) As stated in class, protons also have spin $s = 1/2$; we can say that the "spin" of this hydrogen atom is the total angular momentum of all the components. What are the allowed total spin quantum numbers of this atom? How many complete sets of states are there for each of those total spins? Argue that you have found the correct total number of states.

2. **Spin Interactions**

   (a) Two spin 1/2 particles are fixed in position but have interacting spins. Their Hamiltonian is

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} \tag{1}$$

   for some constant J. Here $S^{(i)}$ is the spin operator of the ith particle. Find the energy eigenvalues of this system, their degeneracies, and the corresponding eigenstates. *Hint*: You will want to work in terms of the total spin quantum numbers.

   (b) The two spins have the same gyromagnetic ratio γ. In the presence of a magnetic field, the Hamiltonian becomes

   $$H = J\vec{S}^{(1)} \cdot \vec{S}^{(2)} - \gamma\vec{B} \cdot \left(\vec{S}^{(1)} + \vec{S}^{(2)}\right) \ . \tag{2}$$

   Now find the energy eigenvalues and their degeneracies. You may take $\vec{B}$ to lie along the z direction.

3. **Hyperon Decay**

   A spin-3/2 Ω^- hyperon at rest decays into a spin-1/2 Λ hyperon and a spin-0 K^- meson. (These are subatomic particles.)

   (a) What are the allowed values of the total orbital angular momentum quantum number ℓ of the final particles, consistent with conservation of angular momentum? Note that the total spin of the two product particles is $s = 1/2$.

   (b) Assume the Ω^- is in a spin state with S_z eigenvalue $+3\hbar/2$. Write the most general possible combined orbital angular momentum and spin state $|\psi\rangle$ of the final particles. Express your answer as a superposition of basis states of the form $|\ell, m\rangle|s, m_s\rangle$. *Hint:* One or more of the following addition of angular momentum change of basis formulas may be helpful:

   $$\begin{aligned}
   |3/2, 3/2\rangle_{tot} &= |1, 1\rangle_1 |1/2, 1/2\rangle_2 \\
   |3/2, 3/2\rangle_{tot} &= \sqrt{\frac{3}{5}}|3/2, 3/2\rangle_1 |1, 0\rangle_2 - \sqrt{\frac{2}{5}}|3/2, 1/2\rangle_1 |1, 1\rangle_2 \\
   |3/2, 3/2\rangle_{tot} &= \sqrt{\frac{4}{5}}|2, 2\rangle_1 |1/2, -1/2\rangle_2 - \sqrt{\frac{1}{5}}|2, 1\rangle_1 |1/2, 1/2\rangle_2 \ .
   \end{aligned}$$

The left-hand sides of these equations are total angular momentum states, while the subscripts on the right-hand sides indicate the first and second angular momentum states added. (These can be read off from tables of Clebsch-Gordon coefficients.)

(c) Substitute the orbital part of the angular momentum state you found in part (b) above with the appropriate spherical harmonic, where θ, ϕ represent the direction of motion of the K^- particle. Then the normalization of the state can be written as

$$\langle\psi|\psi\rangle = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, \sin\theta \, P(\theta, \phi) \ ,$$

where $P(\theta, \phi)$ is the probability density for the angular distribution of the K^-. Find $P(\theta, \phi)$.