PHYS-4601 Homework 10 Due 26 Nov 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Commutators and Things** *partly from Griffiths 4.19, partly inspired by problems in Ohanian*
   (a) Find the commutators $[L_z, x]$, $[L_z, y]$, $[L_z, p_x]$, and $[L_z, p_y]$.
   (b) Show that $\langle x \rangle = 0$ and $\langle p_x \rangle = 0$ for any eigenstate of L_z.
   (c) Show that $[L_z, H] = 0$ for Hamiltonian $H = \vec{p}^2/2m + V$ with central potential V. Argue that therefore $[\vec{L}^2, H] = 0$ also. *Hint:* Assume $V(r)$ is a power series; then argue L_z and r commute based on $[L_z, r^2]$.
   (d) Find the uncertainty relation for the operators L_z and $\sin\phi$.

2. **Probabilities and Expectations** *based on problems by Ohanian*

   Some particle has a wavefunction

$$\psi(\vec{x}) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\sin^2\theta\left[1 + \sqrt{14}\cos\theta\right]\cos(2\phi)R(r) \ , \tag{1}$$

   where R is normalized ($\int_0^\infty dr\, r^2|R|^2 = 1$).
   (a) If you measure the total orbital angular momentum $\vec{L}^2$, what are the possible values you would find and the probabilities you would measure each value?
   (b) Find $\langle L_x \rangle$, $\langle L_y \rangle$, and $\langle L_z \rangle$ in this state.
   (c) Find the uncertainty of L_z.

3. **Angular Dependence of Spherical Harmonic Oscillator**

   Consider a 3D isotropic harmonic oscillator (potential $V(\vec{x}) = (1/2)m\omega^2 r^2$). Recall from a previous assignment that separation of variables in Cartesian coordinates allows us to write the eigenstates as $|n_x, n_y, n_z\rangle = (1/\sqrt{n_x!n_y!n_z!})(a_x^\dagger)^{n_x}(a_y^\dagger)^{n_y}(a_z^\dagger)^{n_z}|0\rangle$.
   (a) Write the wavefunctions for the three states with one excitation in Cartesian coordinates. That is, find $\langle\vec{x}|1,0,0\rangle$, $\langle\vec{x}|0,1,0\rangle$, and $\langle\vec{x}|0,0,1\rangle$.
   (b) Convert those wavefunctions into spherical coordinates and write them as superpositions of terms of the form (radial function) $\times$ (spherical harmonic).
   (c) Write L_z in terms of the Cartesian harmonic oscillator ladder operators ($a_x, a_x^\dagger$, etc). Then use ladder operator techniques to show that $(|1,0,0\rangle \pm i|0,1,0\rangle)/\sqrt{2}$ and $|0,0,1\rangle$ are eigenstates of L_z and find their eigenvalues. Verify that your answers are consistent with the wavefunctions you found in part (b).