# Quantum Mechanics II PHYS-4601
# First In-Class Test

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## Instructions:

- Do not turn over until instructed.
- You will have 75 minutes to complete this test.
- No electronic devices or hardcopy notes are allowed.
- INSTRUCTIONS REGARDING TEST LENGTH WILL GO HERE.
- **Answer all questions briefly and completely.**
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Formulae:

- Schrödinger Equation
  - time-dependent and position-basis time-independent

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle\ ,\quad \langle\vec{x}|H|\psi\rangle = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{x}) + V(\vec{x})\psi(\vec{x}) = E\psi(\vec{x})$$

  - Ehrenfest's Theorem and time-evolution operator

$$\frac{d\langle\mathcal{O}\rangle}{dt} = \frac{i}{\hbar}\langle[H,\mathcal{O}]\rangle + \left\langle\frac{\partial\mathcal{O}}{\partial t}\right\rangle\ ,\quad e^{-iHt/\hbar}$$

- Momentum and Position
  - Inner product of eigenstates (change of basis) in 1D

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}\ \text{ or }\ |p\rangle = \int dx\,\frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}|x\rangle$$

    Normalization is given for infinite volume
  - Operator commutator $[x,p] = i\hbar$
  - Operator representation in position basis

$$\langle x|p|\psi\rangle = -i\hbar\frac{d\psi}{dx}\ \Leftrightarrow p \simeq -i\hbar\frac{d}{dx}$$

- Uncertainty
  - The uncertainty $\sigma_{\mathcal{O}}$ obeys $\sigma_{\mathcal{O}}^2 = \langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2$
  - Heisenberg Uncertainty Prinicple $\sigma_A\sigma_B \geq (1/2)|\langle[A,B]\rangle|$

- Delta Functions in Potentials $V(x) = -\alpha\delta(x-a) + \cdots$

$$\lim_{\epsilon\to 0}\left(\frac{d\psi}{dx}(a+\epsilon) - \frac{d\psi}{dx}(a-\epsilon)\right) = -\frac{2m\alpha}{\hbar^2}\psi(a)$$

- Infinite Square Well $V(x) = 0$ for $-a < x < a$, $V(x) = \infty$ otherwise
  - Eigenfunctions

$$\psi_n(x) = \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right)\ (n\text{ odd) or }\psi_n(x) = \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right)\ (n\text{ even})$$

  - Energy Eigenvalue $E_n = (\hbar^2/2m)(n\pi/2a)^2$
- Probability current $\vec{j} = (i\hbar/2m)\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right)$
- 1D Harmonic Oscillator $V(x) = (1/2)m\omega^2x^2$
  - Stationary State Wavefunctions (see Hermite polynomials below)

$$\langle x|n\rangle = \psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{m\omega}{\hbar}}\,x\right)\exp\left[-\frac{m\omega x^2}{2\hbar}\right]$$

  - Ladder operators

$$a = \sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2\hbar m\omega}}\ ,\ a|n\rangle = \sqrt{n}|n-1\rangle\ ,\ a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle\ ,\ [a, a^\dagger] = 1$$

  - Hamiltonian $H = \hbar\omega\left(a^\dagger a + 1/2\right)$
- Pure math
  - Integrals

$$\int_0^\infty dx\, x^p e^{-x/b} = p!b^{p+1}\ (b > 0),\quad \int_{-\infty}^{\infty} dx\, e^{-ax^2+bx} = \sqrt{\frac{\pi}{a}}e^{b^2/4a}\ (\mathrm{Re}\, a > 0)$$

  - Trig and hyperbolic trig functions

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}\ ,\ \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}\ ,\ \cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}\ ,\ \sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

  Derivatives, integrals, and angle-addition formulae follow from these

**TABLE 2.1:** The first few Hermite polynomials, $H_n(\xi)$.

$H_0 = 1,$
$H_1 = 2\xi,$
$H_2 = 4\xi^2 - 2,$
$H_3 = 8\xi^3 - 12\xi,$
$H_4 = 16\xi^4 - 48\xi^2 + 12,$
$H_5 = 32\xi^5 - 160\xi^3 + 120\xi.$