PHYS-3301 Winter Homework 8 Due 9 Mar 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **A Moving Object** *Barton 6.6 plus*

   A relativistic car moves along the x axis, passing $x = 0$ at $t = 0$. Its velocity is $u(t) = dx/dt = c/\cosh(\omega t)$. **Important:** In the following, you will find some integrals involving hyperbolic trig functions. You may use computer programs such as Maple or Mathematica to do them ***only*** if you *cite the program you use* and *check the result of the indefinite integral by differentiating it and seeing that you get the integrand.*

   (a) Sketch the worldline of the car on a spacetime diagram. Can a real car follow this path exactly?

   (b) Find the relation between the time t and the car's proper time τ. Choose integration constants so that $\tau = 0$ when $t = 0$. Be careful taking signs if you take a square root.

   (c) Since the car moves, it doesn't experience as much proper time as coordinate time t. Find the total time lag Δ over all time, which is defined as

   $$\Delta = \lim_{t\to\infty} [t - \tau(t)] - \lim_{t\to-\infty} [t - \tau(t)] \ .$$

2. **Ultrarelativistic Velocity Addition** *from Hogg 4.8*

   A neutral pion particle (π^0) of mass M is produced at rest with respect to the lab frame. In one possible but rare decay, it produces an electron and positron (anti-electron) which move off in opposite directions, each with mass m and relativistic γ factor of $\gamma = M/2m \approx 100$.

   (a) Since γ is so large, the speed of the electron or positron relative to the lab can be written as $u/c = 1 - \epsilon$. Find ϵ to the lowest order in the small number m/M (that is, if ϵ is written as a power series in m/M, find the power series out to the lowest power with a nonzero coefficient) and then to 1 significant digit.

   (b) Relative to the electron, what is the positron's speed? Again, write the relative speed as $u/c = 1 - \tilde{\epsilon}$ and find $\tilde{\epsilon}$ to lowest order in m/M and to 1 significant digit.

3. **Gene and George**

   George Lucas and Gene Roddenberry decide to race each other from Earth to Mars over a distance d. Lucas travels in a straight line from Earth to Mars at constant speed, while Roddenberry travels along a semicircle of radius $d/2$ at constant speed. Roddenberry's ship is faster, though, so both arrive at Mars at the same time ($t = T$ in the common rest frame of Earth and Mars). Label Earth's position as $x = -d/2$ and Mars's position as $x = d/2$.

   (a) Who has aged more during the trip? Explain your answer qualitatively.

   (b) Assuming our two racers leave Earth at $t = 0$ in the Earth-Mars frame, write Roddenberry's position and velocity as functions of time t and his proper time τ. Assume that Roddenberry moves in the (x, y) plane.

   (c) What is Roddenberry's velocity in Lucas's rest frame? Write your answer as a function of Roddenberry's proper time. Is this the velocity of circular motion?