# PHYS-3301 Winter Homework 7 Due 2 Mar 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Space Race** *Inspired by Barton 6.1*

   George Lucas has two spaceships. Without entering hyperspace, his $\times$-wing can fly at speed $c/4$, and "The Fastest Hunk of Junk in the Galaxy" (or TFHoJitG for short) can fly at $c/2$. But he wants to race them anyway, over a distance of 10 light-minutes, giving the $\times$-wing a head start. Three things happen in the race:

   A. The $\times$-wing takes off at the starting point.
   B. A while later, TFHoJitG takes off at the starting point.
   C. They arrive at the finish line at the same time.

   (a) Find the proper times between events $A$ and $B$, events $A$ and $C$, and events $B$ and $C$.

   (b) Assume the clocks on board the ships are synchronized at the start of the race and that they accelerate essentially instantaneously. What is the difference between the clock readings on the $\times$-wing's clock and TFHoJitG's clock at the end of the race?

   (c) Draw a spacetime diagram that shows the worldlines of both ships from the reference frame of the fixed starting and ending points. Also show the worldlines of the starting and ending points. Label the worldlines clearly (you may use colors and a legend) and draw the axes perpendicular to each other.

   (d) Now draw a spacetime diagram in the reference frame of the $\times$-wing. Again show the worldlines of the two ships, starting point, and end point, and label them clearly. Again, draw the space and time axes perpendicularly to each other.

   *Note:* A light-minute is the distance light can travel in one minute (in a vacuum).

2. **SpaceKid**

   In the following questions, you may *not* use Lorentz transformations; use the invariant interval instead.

   (a) SpaceKid travels from Earth to alpha Centauri (4 lightyears away) and arrives 5 years later according to clocks on earth. With a revised flight plan, SpaceKid then travels 6 lightyears away to Barnard's star, arriving again 10 years later according to earth clocks. How much has SpaceKid aged during the journey?

   (b) In a separate incident, SpaceKid has to deliver an unstable medicine from alpha Centauri to Wolf 359, which is 8 lightyears away and at rest in the same reference frame. Unfortunately, too much of the medicine will decay after 7 years. SpaceKid hops on his/her fastest ship at alpha Centauri and arrives at Wolf 359 10 years later in the common time of the two stars. Did enough medicine arrive at Wolf 359? Make sure to explain your calculations.

3. **Lightcone Coordinates**

   We are used to labeling time $t$ as distinct from space $x, y, z$. But the Lorentz transformations tell us that there is less difference than we think, so we might try some other set of labels. In this problem, define the lightcone coordinates

$$x_+ = x + ct \ , \quad x_- = -x + ct \ . \tag{1}$$

We can use these as the independent variables to describe physics if we want, writing $x$ and $t$ as functions of $x_+$ and $x_-$.

(a) On a spacetime diagram with the $x$ and $t$ axes perpendicular, draw axes for $x_+$ and $x_-$. Why are $x_+$ and $x_-$ called lightcone coordinates (you should see why on the diagram)?

(b) Find the formula for the invariant interval $\delta s^2$ in terms of $\delta x_+$ and $\delta x_-$.

(c) Consider a reference frame $S'$ moving at speed $v$ in standard configuration with our original reference frame $S$. Define new lightcone coordinates

$$x'_+ = x' + ct' \ , \quad x'_- = -x' + ct' \ . \tag{2}$$

Use the Lorentz transformations to find $x'_+$ and $x'_-$ in terms of $x_+$ and $x_-$.