PHYS-3301 Winter Homework 5 Due 10 Feb 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Angular Momentum** *Barton 2.9 plus*

   Angular momentum for a single particle is defined as $\vec{L} = \vec{x} \times \vec{p}$ in a given reference frame S.

   (a) Show that the value $\vec{L}'$ of the angular momentum measured in a reference frame S' is

   i. $\vec{L}' = \vec{L} - \vec{b} \times \vec{p}$ if S' is translated by $\vec{b}$ compared to S. (Meaning that angular momentum does depend on your choice of origin.)

   ii. $\vec{L}' = \vec{L} + \vec{v} \times (m\vec{x} - \vec{p}t)$ if S' is boosted by $\vec{v}$ compared to S.

   (b) Define the total angular momentum $\vec{\mathbf{L}} = \sum_i \vec{L}_i$ as the sum of the individual particle angular momenta in a system. Suppose $\vec{\mathbf{L}}$ is measured in the CM frame of the system with the origin chosen at the position of the center of mass. Show that the total angular momentum $\vec{\mathbf{L}}'$ measured in any frame S' that is translated and boosted with respect to S is equal to $\vec{\mathbf{L}} + M\vec{b} \times \vec{v}$, where M is the total mass.

2. **Swimming Up-River** *based on Hogg 1-4*

   You are on the west bank of a south-flowing river. The river flows at speed v past the ground, and you are able to swim at speed u in still water with $u > v$. Suppose you want to swim and land due east of your current location on the opposite bank of the river.

   (a) At what angle to the west-east axis must you aim yourself (and is it to the north or south)? Note: even though you are aiming differently, you end up swimming due east with respect to the ground.

   (b) What is your speed relative to the ground if you swim due east as described above?

   (c) Explain briefly how this calculation relates to (part of) the Michelson-Morley experiment.

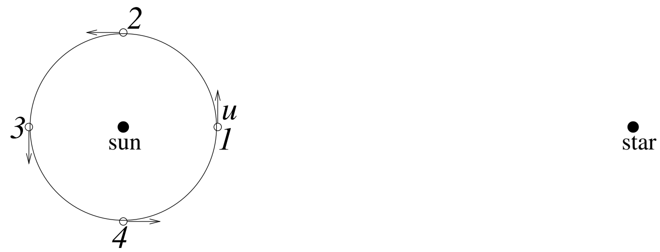
3. **Stellar Aberration**

   In this problem we will explore more the aberration of starlight that was measured as far back as 1725. In this problem, all speeds of objects are small compared to the speed of light, so you are free to use Newtonian/Galilean relativity. You may want to recall that the speed of light is approximately $c = 3 \times 10^8$ m/s.

   (a) First, to get a feel for how this works, consider the following situation. You're driving in a car, and it's raining. Relative to the fixed earth, the rain falls straight down with speed w, and you drive at speed u. At what angle from the vertical do you see the rain falling?

   (b) Now, suppose there is a star straight overhead compared to your telescope. However, the earth is at position 1 in its orbit around the sun (see the figure below), where the orbital speed of the earth is approximately $u = 30,000$ m/s. At what angle from the vertical must I, standing on the earth, aim my telescope so that light from the star falls down the telescope tube? You may ignore the rotational speed of the earth's surface, which is much smaller than the earth's orbital speed. *Hint:* Recall that $\tan\theta \approx \sin\theta \approx \theta$ for small angles

θ.



Note that the figure is not to scale; the star is far enough away that it is effectively directly overhead (at the appropriate time of day) no matter where the earth is in its orbit. Give the angle in arc-seconds, where 3600 arcsec equal 1 degree.

(c) At which point(s) as labeled in the figure above is this angle of aberration maximized? At which points is it minimized?