

PHYS-3301 Winter Homework 2 Due 27 Jan 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. How to Calculate Gaussian Integrals

- (a) Start by evaluating the integral

$$J(a) \equiv \int_0^{\infty} dx x e^{-ax^2} . \quad (1)$$

Hint: it's probably easiest to use the substitution $u = x^2$.

- (b) Evaluate the integral

$$I(a) \equiv \int_0^{\infty} dx e^{-ax^2} \quad (2)$$

as follows:

- i. Write

$$(I(a))^2 = \int_0^{\infty} dx \int_0^{\infty} dy e^{-ax^2} e^{-ay^2} \quad (3)$$

(be careful to rename dummy integration variables) and then convert to plane polar coordinates.

- ii. You should now be able to use your result for $J(a)$ to calculate $(I(a))^2$. Then find $I(a)$. Your answer should be $I(a) = (1/2)\sqrt{\pi/a}$.

- (c) Calculate the integral

$$\int_0^{\infty} dx x^2 e^{-ax^2} \quad (4)$$

by showing first that it is equal to $-dI(a)/da$.

2. The Ideal Gas

A classical ideal gas is composed of molecules of mass m with the number of molecules per energy level given by the Maxwell-Boltzmann distribution for distinguishable particles. As discussed in class, the number of particles in a state of energy E_k is

$$n(E_k) = e^{(\mu - E_k)/k_B T} \text{ where } E_k = \frac{\hbar^2 k^2}{2m} , \quad (5)$$

and the degeneracy at energy E_k is $d_k = (V/2\pi^2)k^2 dk$, as discussed in the class notes.

- (a) Using your results from the previous problem, show that the number of air molecules is given by $N/V = (mk_B T/2\pi\hbar^2)^{3/2} \exp(\mu/k_B T)$.
- (b) Using your results from the previous problem, show that the total energy is $3Nk_B T/2$.

3. Cold Electron Gas

Electrons confined in a piece of metal can also be described as particles in a 3D infinite square well. They have 2 spin states, so their energy levels have twice the degeneracy of an ideal gas $d_k = (V/\pi^2)k^2 dk$. They follow the Fermi-Dirac distribution; as $T \rightarrow 0$, this distribution turns in to a step function that is equal to 1 for $E_k < \mu$ and 0 for $E_k > \mu$. Find the relationships between the number N of electrons and μ and between total energy E and μ .

4. **Number of Photons** *based on Scherrer 1.10*

Now we consider photons following the Bose-Einstein distribution as in our discussion of the Planck law in class. Integrate the Bose-Einstein distribution times the degeneracy factor to show that the total number density of photons at temperature T is $N/V = \alpha(kT/hc)^3$, where α is a numerical constant (you do not need to evaluate α but can leave it in the form of a dimensionless integral.).