PHYS-3301 Winter Homework 12

This homework is **NOT TO BE HANDED IN OR GRADED**. It is for studying and preparation for the final exam. Solutions will be posted on April 7.

1. **Reflected Sound** *based on Barton*

   Consider a police car situated to the left of a large tractor-trailer truck. The sound of the police siren reflects off the back of the truck and back to the police officer. In the police car's frame, the initial sound frequency is ω. In the two situations below, what is the frequency of the reflected wave that the police officer hears? *Hint:* In the rest frame of the reflecting surface, the reflected sound wave has the same frequency as the incident sound wave.

   (a) In a frame with no wind, the police car moves at speed $u < c_s$ toward the truck, which is stationary.

   (b) In a frame with no wind, the truck moves at speed $u < c_s$ toward the police car, which is stationary. If your answer is different than the previous part, explain.

2. **Spaceship Communications**

   Space station ES10 sits at galactic coordinates $x = 0$ lyr, $y \equiv y_0 = 3$ lyr. The station is able to receive radio signals in the range of $\omega_1 = 3$ MHz to $\omega_2 = 25/3$ MHz. Suppose that the experimental starship NX-02 travels along the x axis at constant speed $u = 4c/5$, passing $x = 0$ at $t = 0$ according to the spacestation. The starship broadcasts its positions at frequency $\omega_E = 5$ MHz. From what positions x can the space station receive the starship's signal?

3. **The Cosmic Microwave Background**

   The universe is filled with left-over radiation from the Big Bang called the Cosmic Microwave Background, which has a typical frequency of $\bar{\omega} = 160$ GHz averaged over the sky. There is a smooth variation of frequency at different points on the sky. The maximum observed frequency is 200 MHz higher than average in one particular spot on the sky; the minimum frequency is 200 MHz lower than average at the diametrically opposite point. This effect is due to the motion of our galaxy relative to the overall rest frame of the microwave background.

   (a) What is the relative speed of the earth and the source of the Cosmic Microwave Background in km/s? Remember that $c \approx 3 \times 10^8$ m/s.

   (b) Suppose we measure the frequency at an angle θ on the sky from the maximum frequency point. Show that the frequency is

   $$\omega(\theta) = \bar{\omega} + (200\ \text{MHz}) \cos\theta \ . \tag{1}$$

4. **Fast Orbit**

   The star S-2 orbits the (presumed) black hole at the center of our galaxy with the highest known orbital speed of any astrophysical object. Consider a simplified model of its orbit as a perfect circle with a period of $T \approx 15$ yr and orbital speed of $u \approx 0.02c$. Also suppose that the earth is in the same plane as S-2's orbit at a distance much greater than the size of the orbit. S-2 emits a spectral line of frequency $\omega \approx 8 \times 10^{14}$ Hz; what is the difference of the observed frequency at earth to the emitted frequency as a function of emitted time t? Let $t = 0$ be the distance of closest approach between S-2 and earth. Give the formula first in terms of variables and then calculate numerical values. Work to 1 significant figure. You may safely assume that the earth's orbital speed around the sun and the sun's orbital speed around the center of the galaxy are negligible.