PHYS-3301 Winter Homework 11 Due 30 Mar 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **GZK Cut-off**

   The universe is filled with photons left over from the Big Bang which have a typical energy 2.5×10^{-4} eV. This is called the cosmic microwave background (CMB) radiation. Suppose a proton of energy E and mass m_p hits a typical CMB photon head-on. What is the minimum value of E required to convert the photon to a pion of mass m_π? In other words, how large must E be for the reaction $p + \gamma \to p + \pi^0$ to occur? Use $m_p = 938$ MeV$/c^2$ and $m_\pi = 135$ MeV$/c^2$ and assume that $m_p \ll E$. FYI, Greisen, Zatsepin, and Kuzmin predicted that protons of higher energies cannot travel for extremely long distances through the universe because they lose energy to pion production in this way.

2. **Inverse Compton Scattering**

   A photon of energy ϵ strikes an electron of energy E and mass m head on (that is, the spatial parts of their momenta are opposite each other). Call the initial 4-momentum of the photon q^μ and of the charged particle p^μ.

   (a) Suppose that the final photon moves back along the original photon's path. Find the energy ϵ' of the final photon.

   (b) Suppose E is very large, so you can ignore the electron mass. Use your answer from part (a) to show that $\epsilon' = E$ in this limit. (This shows that "inverse Compton scattering" can increase photon energies, since the photon steals essentially all the electron's energy.)

3. **Large Hadron Collider**

   The Large Hadron Collider (LHC) collides pairs of protons with a total energy of about 10 TeV ($= 10^4$ GeV) in their CM frame. For this problem, you will want to know that the mass of a proton is about 1 GeV$/c^2$.

   (a) What is the energy of one of the protons as measured in the rest frame of the other proton? *Hint:* First show that $(p_1 + p_2)^2$, where p_1^μ, p_2^μ are the 2 initial 4-momenta, gives the total CM frame energy. Then show that the relativistic scalar product $p_1 \cdot p_2$ gives the energy of the first proton in the 2nd proton's rest frame and relate the two products.

   (b) Suppose the LHC observes a collision that produces two photons and a number of other particles. The two photons have energies of 0.9 TeV and 0.6 TeV, and their paths are at an angle of 60 degrees to each other. If the collision process can be described as 2 protons become stuff plus particle X, followed by particle X decaying into the 2 photons, what is the mass of particle X?

4. **Relativistic Newton's Cradle**

   A particle of type B smashes into a particle of type C, turning into one particle of type A and one of type D, with masses $m_A < m_B < m_C < m_D$. Suppose that the C particle is intially at rest and the D particle is produced also at rest. (This can really happen when the B particle is a kaon, the C is a proton, the D is a lambda hyperon, and the A is a pion.)

   (a) What is the initial energy of the B particle?

   (b) What is the final energy of the A particle?