PHYS-3301 Winter Homework 10 Due 23 Mar 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Rocket Car**

   Suppose a rocket car drives along the x axis with velocity $u = dx/dt = \alpha t/\sqrt{1+\alpha^2 t^2/c^2}$ in the lab frame.

   (a) Find relationship between the coordinate time t and the car's proper time τ and the components of the car's 4-velocity U^μ as a function of its proper time τ.

   (b) Find the car's position $x(\tau)$ as a function of its proper time. Choose integration constants so $x(0) = 0$.

   (c) Define the *4-acceleration* (aka *proper acceleration*) $A^\mu \equiv dU^\mu/d\tau$. Find the components A^μ and the square $A^2 = A_\mu A^\mu$. Also show that $A_\mu U^\mu = 0$.

2. **Lorentz Force**

   Recall from the previous assignment that the electric and magnetic fields can be written as an antisymmetric relativistic tensor with two indices $F^{\mu\nu}$. The independent components are (here, $i = 1, 2, 3$ is a space index)

$$F^{0i} = E^i \ , \quad F^{12} = B^3 \ , \quad F^{13} = -B^2 \ , \quad F^{23} = B^1 \ . \tag{1}$$

   Since $F^{\mu\nu}$ is antisymmetric, the diagonal components $F^{00} = F^{11} = F^{22} = F^{33} = 0$. (We have chosen a convenient system of units where the electric and magnetic field have the same dimension.) Show that the covariant equation

$$\frac{dp^\mu}{d\tau} = \frac{q}{c} U_\nu F^{\mu\nu} \tag{2}$$

   reproduces the Lorentz force law in the x direction

$$\frac{dp^1}{dt} = qE^1 + \frac{qu^2}{c} B^3 - \frac{qu^3}{c} B^2 \ , \tag{3}$$

   for a particle of charge q, coordinate velocity u^i, and 4-velocity U^μ.

3. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

$$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 \ . \tag{4}$$

Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

(b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E. Use (4).

(c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.

For your interest, these neutrino measurements were made by a predecessor experiment to one of the experiments that led to the 2015 Nobel Prize in Physics.

4. **Energy Release Efficiencies**

In this question, you will find the fraction of mass energy released in a couple of nuclear reactions. It will be useful for you to know that one atomic mass unit (aka one *dalton*) is 930 MeV/c^2 and also 1.7×10^{-27} kg (both to two significant figures).

(a) In a nuclear reactor, a slow (nonrelativistic) neutron of mass 940 MeV/c^2 hits a Uranium-235 nucleus (mass 240 dalton), which breaks into various fragments. This *fission* (splitting) reaction releases 200 MeV of energy into heat, which the reactor can then convert to work. What fraction of the initial mass energy is converted to useful heat?

(b) The sun produces heat through the *fusion* of light nuclei into heavier ones. Most of this "hydrogen burning" effectively converts 4 protons (mass 940 MeV/c^2) into a He-4 nucleus (mass 4.0 dalton), 2 positrons (aka anti-electrons) of mass 0.51 MeV/c^2, two neutrinos (essentially zero mass), and photons (zero mass). The two positrons immediately find two electrons and annihilate into photons. Assume that all of the energy above the rest energy of the final state particles is converted to heat (we ignore the few percent of energy that is carried away by the neutrinos). What fraction of the initial mass energy is converted to heat?