PHYS-3301 Winter Homework 1 Due 13 Jan 2016

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

Helpful Formulae: You might find the integral

$$\int_0^\infty dx\, x^n e^{-x/a} = n!\, a^{n+1} \quad (a > 0,\ n = 0, 1, 2, \cdots) \tag{1}$$

useful. You will also want to know that the angular wavefunctions satisfy

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta\, \sin\theta\, |Y_\ell^m(\theta, \phi)|^2 = 1\ . \tag{2}$$

1. **Ground State Radii**

   For this problem, consider the ground state of hydrogen as described by the wavefunction given in table 6.2 of Scherrer (or the formula in the text just above that table). Give your answers in terms of the Bohr radius.

   (a) Find expectation value of the radius $\langle r \rangle$ for this state.

   (b) Find the root-mean-square (rms) radius $r_{rms} \equiv \sqrt{\langle r^2 \rangle}$ for this state.

   (c) Find the most probable radius $\bar{r}$ for this state. $\bar{r}$ is defined as the value such that

   $$\lim_{\Delta \to 0} \int_0^{2\pi} d\phi \int_0^\pi d\theta\, \sin\theta \int_{\bar{r}}^{\bar{r}+\Delta} dr\, r^2 |\psi(r, \theta, \phi)|^2 \tag{3}$$

   is maximized.

2. **High Angular Momentum States**

   In this question, consider an electron in a state of hydrogen with the largest possible value of angular momentum $\ell = n - 1$. From our class notes, we know that the radial wavefunction can be written as

   $$R(r) = A r^{n-1} e^{-r/na_0}\ . \tag{4}$$

   (a) Find the normalization constant A.

   (b) Find $\langle r^2 \rangle$ in this state.