PHYS-3301 Homework 9 Due 12 Nov 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **GZK Cut-off**

   The universe is filled with photons left over from the Big Bang which have a typical energy 2.5×10^{-4} eV. This is called the cosmic microwave background (CMB) radiation. Suppose a proton of energy E and mass m_p hits a typical CMB photon head-on. What is the minimum value of E required to convert the photon to a pion of mass m_π? In other words, how large must E be for the reaction $p + \gamma \to p + \pi^0$ to occur? Use $m_p = 938$ MeV/c^2 and $m_\pi = 135$ MeV/c^2 and assume that $m_p \ll E$. FYI, Greisen, Zatsepin, and Kuzmin predicted that protons of higher energies cannot travel for extremely long distances through the universe because they lose energy to pion production in this way.

2. **Inverse Compton Scattering**

   A photon of energy ϵ strikes an electron of energy E and mass m head on (that is, the spatial parts of their momenta are opposite each other). Call the initial 4-momentum of the photon q^μ and of the charged particle p^μ.

   (a) Suppose that the final photon moves back along the original photon's path. Find the energy ϵ' of the final photon.

   (b) Suppose E is very large, so you can ignore the electron mass. Use your answer from part (a) to show that $\epsilon' = E$ in this limit. (This shows that "inverse Compton scattering" can increase photon energies, since the photon steals essentially all the electron's energy.)

3. **3-Body Kaon Decay** *essentially Barton 11.3(a)*

   A K particle (*aka* a "kaon") of mass $M = 494$ MeV/c^2 can decay into three identical π particles ("pions") of mass $m = 140$ MeV/c^2. Suppose that a resting kaon decays with one of the final pions also at rest. First, argue that the other two pions have the same energy. Then find that energy (in terms of m and M and in terms of MeV).

4. **Tachyons**

   Lorentz symmetry is actually consistent with particles, called *tachyons*, that travel faster than the speed of light ("tachy" is a Greek root meaning "fast").

   (a) Define the tachyonic 4-momentum with the energy and spatial momentum in the usual components such that $p_\mu p^\mu = +m^2c^2$ (*the opposite sign from usual*). Find the speed v as a function of energy and the tachyonic mass m and show that $v \to c$ as $E \to \infty$. Assume the usual relationship $v/c = |\vec{p}|/p^0$.

   (b) Suppose an electron — a *normal* massive particle – with mass M — starts at rest and emits a tachyon with tachyonic mass m (so the final state is an electron plus a tachyon). Find the final tachyon and electron energies and show that the electron *gains* energy. This is an example of why tachyons are considered instabilities in particle physics; they can cause "spontaneous acceleration" of particles.