PHYS-3301 Homework 8 Due 5 Nov 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Lorentz Force**

   Recall from the previous assignment that the electric and magnetic fields can be written as an antisymmetric relativistic tensor with two indices $F^{\mu\nu}$. The independent components are (here, $i = 1, 2, 3$ is a space index)

   $$F^{0i} = E^i \ , \quad F^{12} = B^3 \ , \quad F^{13} = -B^2 \ , \quad F^{23} = B^1 \ . \tag{1}$$

   Since $F^{\mu\nu}$ is antisymmetric, the diagonal components $F^{00} = F^{11} = F^{22} = F^{33} = 0$. (We have chosen a convenient system of units where the electric and magnetic field have the same dimension.) Show that the covariant equation

   $$\frac{dp^\mu}{d\tau} = \frac{q}{c} U_\nu F^{\mu\nu} \tag{2}$$

   reproduces the Lorentz force law in the x direction

   $$\frac{dp^1}{dt} = qE^1 + \frac{qu^2}{c} B^3 - \frac{qu^3}{c} B^2 \ , \tag{3}$$

   for a particle of charge q, coordinate velocity u^i, and 4-velocity U^μ.

2. **Energy Release Efficiencies**

   In this question, you will find the fraction of mass energy released in a couple of nuclear reactions. It will be useful for you to know that one atomic mass unit (aka one *dalton*) is 930 MeV/c^2 and also 1.7×10^{-27} kg (both to two significant figures).

   (a) In a nuclear reactor, a slow (nonrelativistic) neutron of mass 940 MeV/c^2 hits a Uranium-235 nucleus (mass 240 dalton), which breaks into various fragments. This *fission* (splitting) reaction releases 200 MeV of energy into heat, which the reactor can then convert to work. What fraction of the initial mass energy is converted to useful heat?

   (b) The sun produces heat through the *fusion* of light nuclei into heavier ones. Most of this "hydrogen burning" effectively converts 4 protons (mass 940 MeV/c^2) into a He-4 nucleus (mass 4.0 dalton), 2 positrons (aka anti-electrons) of mass 0.51 MeV/c^2, two neutrinos (essentially zero mass), and photons (zero mass). The two positrons immediately find two electrons and annihilate into photons. Assume that all of the energy above the rest energy of the final state particles is converted to heat (we ignore the few percent of energy that is carried away by the neutrinos). What fraction of the initial mass energy is converted to heat?

3. **Speed at High Energies**

   The next two problems will deal with particles that have energy much greater than their rest mass energies, $E \gg mc^2$. Show that such a highly energetic particle has a speed approximately given by

   $$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 \ . \tag{4}$$

Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

4. **Energy Loss to Radiation in Circular Motion**

   Protons in the Large Hadron Collider (LHC) experiment move in a circular path of radius R and constant energy E. Unfortunately, accelerating charged particles like protons radiate energy away, so there is a constant need to add energy to keep the protons up to speed. In this problem, you will find the rate at which protons lose energy to radiation. Take the values $R = 10$ km, $E = 10$ TeV=10000 GeV, and the proton mass $mc^2 = 1$ GeV. Based on the result of problem 3 above, you may approximate the speed of a proton in the LHC by c (but you will need to find the precise value for the gamma factor).

   (a) Assuming the proton moves in a counter-clockwise direction in the xy plane, write the components of its 4-velocity U^μ as a function of proper time τ. Give your answer in terms of E, m, R, and constants, *not* numerical values. *Hint:* You may want to start by writing U^μ as a function of lab frame time t and then convert t to τ.

   (b) The power loss of the proton in the LHC frame is given by the *Larmor formula*

   $$P = \frac{2}{3c^3} \frac{e^2}{4\pi\epsilon_0} \frac{dU_\mu}{d\tau} \frac{dU^\mu}{d\tau} \ . \tag{5}$$

   Use your results to find the power radiated by a proton at the LHC in SI units. If there are 3×10^{14} protons in the LHC at any time, how much power is needed to keep all the protons up to speed? Use the value $e^2/4\pi\epsilon_0 = 2 \times 10^{-28}$ Jm.

5. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   (a) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E. Use (4).

   (b) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.