# PHYS-3301 Homework 4 Due 8 Oct 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Twins** *From Barton 4.4 and about everywhere else*

   Frankie and Fannie are twins, born on earth at essentially the same time. At birth, Fannie is loaded onto a starship that travels to QY Aurigae, a star 20 lightyears away, at constant speed $u$. On arrival, the starship turns around (essentially immediately) and returns to earth at constant speed $u$. When the ship arrives back at earth, Fannie is 30 years old (clearly, this is a fast starship). Let's assume that Fannie isn't killed or otherwise maimed by any accelerations on the starship.

   (a) In any (correct) manner you choose, find the speed $u$ of the starship as a fraction of light speed and Frankie's age when Fannie arrives back at earth.

   (b) The earth and QY Aurigae are at positions $x = 0$ lightyears and $x = 20$ lightyears in Frankie's inertial reference frame. Write Fannie's path as a function $x(t)$, where $t$ is measured in Frankie's reference frame.

   (c) Every ten years on his birthday, Frankie sends a light signal to Fannie. Does Fannie receive any of the signals before she reaches the star and turns around?

   (d) Draw a spacetime diagram in the Frankie's reference frame. Show Fannie's worldline and the worldlines of the light signals.

   (e) Based on your spacetime diagram, what's different about Frankie and Fannie? That is, why shouldn't they be the same age? *Hint:* why don't we draw a spacetime diagram from Fannie's rest frame?

   *Note:* Light speed can be written in appropriate units as $c = 1$ lightyear/year. Those units might make life easier.

2. **SpaceKid, Part II**

   In the following questions, you may *not* use Lorentz transformations; use the invariant interval instead.

   (a) Remember from assignment 3 that SpaceKid traveled from Earth to alpha Centauri (4 lightyears away) and arrived 5 years later according to clocks on earth. With a revised flight plan, SpaceKid then travels 6 lightyears away to Barnard's star, arriving again 10 years later according to earth clocks. How old is SpaceKid at the end of the journey?

   (b) In a separate incident, SpaceKid has to deliver an unstable medicine from alpha Centauri to Wolf 359, which is 8 lightyears away and at rest in the same reference frame. Unfortunately, too much of the medicine will decay after 7 years. SpaceKid hops on his/her fastest ship at alpha Centauri and arrives at Wolf 359 10 years later in the common time of the two stars. Did enough medicine arrive at Wolf 359? Make sure to explain your calculations.

3. **Lightcone Coordinates**

   We are used to labeling time $t$ as distinct from space $x, y, z$. But the Lorentz transformations tell us that there is less difference than we think, so we might try some other set of labels. In this problem, define the lightcone coordinates

$$x_+ = x + ct \ , \quad x_- = -x + ct \ . \tag{1}$$

We can use these as the independent variables to describe physics if we want, writing $x$ and $t$ as functions of $x_+$ and $x_-$.

(a) On a spacetime diagram with the $x$ and $t$ axes perpendicular, draw axes for $x_+$ and $x_-$. Why are $x_+$ and $x_-$ called lightcone coordinates (you should see why on the diagram)?

(b) Find the formula for the invariant interval $\delta s^2$ in terms of $\delta x_+$ and $\delta x_-$.

(c) Consider a reference frame $S'$ moving at speed $v$ in standard configuration with our original reference frame $S$. Define new lightcone coordinates

$$x'_+ = x' + ct' \ , \quad x'_- = -x' + ct' \ . \tag{2}$$

Use the Lorentz transformations to find $x'_+$ and $x'_-$ in terms of $x_+$ and $x_-$.