

## PHYS-3301 Homework 3 Due 1 Oct 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

### 1. Invariance of Light Speed

In this question, we'll show that the speed of light is invariant, no matter the direction of the velocity. (Of course, our derivation of the Lorentz transformations showed this for light moving along the relative motion of two frames.) Start in the  $S$  frame, where a light beam leaves the origin at time  $t = 0$  and reaches point  $\vec{x} = (x_0, y_0, 0)$  at time  $t = \sqrt{x_0^2 + y_0^2}/c$ , hitting a detector there. Now consider a frame  $S'$  moving at speed  $v$  relative to  $S$  along the  $x$  axis.

- At what coordinates  $t', x', y', z'$  does the light hit the detector? Write your answer in terms of  $c, x_0, y_0$ , and  $v$ .
- Find the components of the light's velocity in the  $S'$  frame by dividing  $x'/t'$ , etc. What is the speed of light in  $S'$ ?
- Why don't we need to think about a component of motion along the  $z$  axis to get a general proof of the invariance of the speed of light?

### 2. Adventures of SpaceKid *somewhat inspired by Barton 4.2 & 4.3*

A young explorer, age 8, leaves earth on an extremely fast rocket.

- He or she travels first to alpha Centauri and arrives 5 years later Earth time and 4 lightyears away by Earth distance. How old is our explorer on arrival at alpha Centauri?
- Immediately after, he or she leaves for epsilon Eridani, arriving 13 years later Earth time at age 16. How far apart are alpha Centauri and epsilon Eridani in the Earth's reference frame?

### 3. Rocket Speed *based on a question by Hartle*

A relativistic rocket of proper length  $L$  passes a space station at speed  $4c/5$ . Some time later, the space station emits a radio signal (light wave), and it arrives at the rocket's tail receivers at some time  $T$  (in the space station's rest frame) later.

- How much later does the radio signal reach the receivers at the nose of the rocket according to clocks on the rocket?
- How much later does the radio signal reach the receivers at the nose of the rocket according to clocks on the space station?
- What is the spatial separation between where the signal reaches the tail of the rocket and where it reaches the nose of the rocket in the space station's frame?